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MATHEMATICS FOR TECHNOLOGY

VOLUME 1

LEARNING MODULE FOR POLYTECHNIC

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MATHEMATICS FOR TECHNOLOGY

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Preface

Thankful to Allah s.w.t for His Bless along the process of completing this module. Congratulations to all lecturers who are involved in drafting, writing and editing this module namely En. Mahadi bin Ripin and En Shakirurahman bin Ismail. We would like to express our gratitude and special thanks to our Head of Department, En. Mohd Nor bin Yusof and appreciate to all of members from Mathematics, Science & Computer Department, Politeknik Jeli Kelantan (PJK) for their support and assistance with numerous materials for this work.

Mathematics For Technology for semester 1 students is a module borne out of the genuine desire to bridge the gap that had existed in this institution for a through and better understanding of the basic tools in Basic Algebra, Geometry, Measurement, Functions & Graph and Statistic for their applications. This module is written the assumptions that students are familiar with first year semester in the respective course are properly covered with many worked examples and exercise given at the end of every chapter to further simplify the understanding of the readers. As an initiative, this module was written to assist in Teaching and Learning process. It also provides step-by-step example followed by exercise for every subtopic. In addition, review questions are attached at the of every chapter as the enhancement for students. The language used is very simple and examples given are easy to understand. In overall, this module is delivered in very simple and comprehensive way.

Thus, we are really hope that this module will be useful for students in enhancing their mathematics skills and developing their interest towards mathematics. It also will be valuable additional reference for lecturers and others.

Thank you

Mohd. Shakirurahman Bin Ismail
Editor

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CHAPTER 1

BASIC ALGEBRA

Written By: En. Mahadi Bin Ripin

BASIC ALGERBRA

1.1 Simplify basic algebra

EXAMPLE

1. Express $\frac{6}{x^2-1} - \frac{3}{x(x-1)}$ in simplest fraction form.

$$= \frac{6}{(x+1)(x-1)} - \frac{3}{x(x-1)}$$

$$= \frac{6\,(x)}{(x+1)(x-1)(x)} - \frac{3\,(x+1)}{x(x-1)(x+1)}$$

$$= \frac{6x-3x-3}{x\,(x+1)(x-1)}$$

$$= \frac{3x-3}{x\,(x+1)\,(x-1)}$$

$$= \frac{3\,(x-1)}{x\,(x+1)(x-1)} = \frac{3\,\cancel{(x-1)}}{x\,(x+1)\cancel{(x-1)}}$$

$$= \frac{3}{x\,(x+1)}$$

2. Express $\frac{x+1}{3} - \frac{x}{x+2}$ in simplest fraction form.

$$= \frac{x+1\,(x+2)}{3\,(x+2)} - \frac{x\,(3)}{x+2\,(3)}$$

$$= \frac{(x+1)(x+2)-3x}{3\,(x+2)}$$

$$= \frac{x^2+3x+2-3x}{3\,(x+2)} = \frac{x^2+3x-3x+2}{3\,(x+2)}$$

$$= \frac{x^2+2}{3\,(x+2)}$$

3. Express $\frac{4}{p-2} + \frac{4+2p}{p(p-2)}$ in simplest fraction form.

$= \frac{4(p)}{(p-2)(p)} + \frac{4+2p}{p(p-2)}$

$= \frac{4p+4+2p}{p(p-2)} = \frac{4p+2p+4}{p(p-2)}$

$= \frac{6p+4}{p(p-2)}$

4. Express $\frac{x^2+2x-15}{x^2-9} \times \frac{x^2+6x+9}{x^2+3x-10}$ in simplest fraction form.

$= \boxed{\frac{(x+5)(x-3)}{(x+3)(x-3)} \times \frac{(x+3)(x+3)}{(x+5)(x-2)}}$ → Factorize the equation

$= \frac{\cancel{(x+5)(x-3)}}{\cancel{(x+3)(x-3)}} \times \frac{\cancel{(x+3)}(x+3)}{(x+5)\cancel{(x-2)}}$

$= \frac{x+3}{x-2}$

5. Express $\frac{x+1}{5y+10} \div \frac{x^2+2x+1}{y+2}$ in simplest fraction form.

$= \frac{x+1}{\boxed{5(y+2)}} \div \frac{\boxed{(x+1)(x+1)}}{y+2}$ → Factorize the equation

$= \frac{x+1}{5(y+2)} \times \frac{(y+2)}{(x+1)(x+1)}$

$= \frac{\cancel{(x+1)}}{5\cancel{(y+2)}} \times \frac{\cancel{(y+2)}}{\cancel{(x+1)}(x+1)}$

$= \frac{1}{5(x+1)}$

EXERCISE 1

Express the following question by solving it in simplest form.

1. $\frac{x^2+x-2}{x^2+4x+3} \times \frac{x^2+2x+1}{x^2+1}$

2. $\frac{2}{x-2} - \frac{4}{x(x-2)}$

3. (a) $\frac{x^2-4}{6} \times \frac{3y}{2x+4}$

   (b) $\frac{2x+4}{3x-6} \div \frac{x^2-4}{x^2+x-6}$

4. $\frac{1}{3x} + \frac{x-6}{12x^2}$

5. $\frac{1}{x(x+1)} - \frac{x}{(x+1)}$

6. $\frac{8xy+3}{15xy^2} + \frac{1}{3y}$

7. $\frac{4x-8}{6} \div \frac{x-2}{3}$

1.2 Show algebraic equations

EXAMPLE

1. Solve the equation.

$$2x + 3 = 4(x + 1)$$

$$2x + 3 = 4(x + 1)$$

$$2x + 3 = 4x + 4$$

$$2x - 4x = 4 - 3$$

$$-2x = 1$$

$$x = -\frac{1}{2}$$

2. Solve the equation.

$$\frac{p-2}{4} - \frac{9-p}{3} = 0$$

$$\frac{p-2}{4} - \frac{9-p}{3} = 0$$

$$\frac{p-2}{4} = \frac{9-p}{3}$$

$$(p-2)(3) = (9-p)(4)$$

$$3p - 6 = 36 - 4p$$

$$3p - 6 = 36 - 4p$$

$$3p + 4p = 36 + 6$$

$$7p = 42$$

$$p = \frac{42}{7} = 6$$

3. Solve the equations.

a) $\frac{5}{2p} = 10$

$$\frac{5}{2p} = 10 \longrightarrow 5 = 10(2p)$$

$$20p = 5 \longrightarrow p = \frac{5}{20}$$

$$p = \frac{\cancel{5}}{\cancel{20}} = \frac{1}{4}$$

b) $3x - (5 - 2x) = 15$

$$3x - 5 + 2x = 15$$

$$3x + 2x = 15 + 5$$

$$5x = 20 \longrightarrow x = \frac{20}{5}$$

$$x = \frac{\cancel{20}}{\cancel{5}} = 4$$

4. Solve the equations.

a) $\frac{1}{2}x - \frac{1}{3} = \frac{1}{3}$

$$\frac{1}{2}x - \frac{1}{3} = \frac{1}{3}$$

$$\frac{1}{2}x = \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{2}x = \frac{2}{3}$$

$$x = \frac{2}{3}(2)$$

$$x = \frac{4}{3}$$

b) $\frac{m-3}{4} = \frac{m}{6}$

$$\frac{m-3}{4} = \frac{m}{6}$$

$$(m-3)(6) = (m)(4) \longrightarrow (m-3)(6) = (m)(4)$$

$$6m - 18 = 4m$$

$$6m - 4m = 18$$

$$2m = 18 \longrightarrow m = \frac{18}{2}$$

$$m = \frac{\cancel{18}}{\cancel{2}} = 9$$

EXERCISE 2

Solve the equations.

1. (a) $\frac{2x}{7} = 1$

   (b) $3p - 2 = 2(p + 3)$

2. (a) $3p = 1 - 2p$

   (b) $\dfrac{x + 2}{3} = x - 6$

3. (a) $2 - 5x = 4(1 - x)$

   (b) $\dfrac{1}{3}(x - 1) = 2$

4. (a) $\frac{2}{m} = 6$

   (b) $9 - (2p + 5) = 4p$

5. (a) $\frac{1}{3}(x + 1) = 3$

   (b) $2(y - 5) - (y - 3) = 2$

6. (a) $2p - \frac{1}{4} = 2\frac{3}{4}$

   (b) $2x + 1 = 5 - 2(2x - 1)$

7. (a) $\frac{x-1}{2} = 5$

   (b) $3(y - 4) = 8 - y$

1.3 Solve simultaneous linear equations

EXAMPLE

1. Solve the following pairs of simultaneous equations by using substitution method.

   $3x - 6y = 24$

   $2x + y = 1$ — 1

   $3x - 6y = 24$ — 2

   $2x + y = 1$ — 3 from equation 2

   $y = 1 - 2x$

   3 into 1

   $3x - 6(1 - 2x) = 24$

   $3x - 6 + 12x = 24 \rightarrow 3x - 6 + 12x = 24$

   $3x + 12x = 24 + 6$

   $15x = 30$

   $x = \frac{30}{15}$

   $x = 2$

   x into 3

   $y = 1 - 2(2)$

   $y = 1 - 4$

   $y = -3$

2. Solve the following pairs of simultaneous equations by using substitution method.

$4x + 9y = 13$

$3x + y = 4$ —— 1

$4x + 9y = 13$ —— 2

$3x + y = 4$ —— 3

$y = 4 - 3x$

3 into 1

$4x + 9(4 - 3x) = 13$ → $4x + 9(4 - 3x) = 13$

$4x + 36 - 27x = 13$ → $4x + 36 - 27x = 13$

$4x - 27x = 13 - 36$

$-23x = -23$ → $-23x = -23$

$x = \frac{-23}{-23}$

$x = 1$

x into 3

$y = 4 - 3(1)$

$y = 4 - 3$

$y = 1$

3. Solve the following pairs of simultaneous equations by using substitution method.

$7x - 2y = 45$

$5x + y = 37$ —— 1

$7x - 2y = 45$ —— 2

$5x + y = 37$ —— 3

$y = 37 - 5x$

3 into 1 $\longrightarrow 7x - 2(37 - 5x) = 45$

$7x - 2(37 - 5x) = 45$ $\longrightarrow 7x - 74 + 10x = 45$

$7x - 74 + 10x = 45$

$7x + 10x = 45 + 74$

$17x = 119$ $\longrightarrow 17x = 119$

$x = \dfrac{119}{17}$

$x = 7$

x into 3

$y = 37 - 5(7)$

$y = 2$

4. Solve the following pairs of simultaneous equations by using elimination method.

$x + y = 14$

$x - y = 8$

$x + y = 14$ —— 1

$x - y = 8$ —— 2

1 − 2

$$x + y = 14$$

$$(-)\,x - y = 8$$

$$2y = 6 \longrightarrow 2\;y = 6$$

$$y = \frac{6}{2}$$

$$y = 3$$

y into 1

$x + y = 14$

$x + (3) = 14 \longrightarrow x + 3 = 14$

$x = 14 - 3$

$x = 11$

5. Solve the following pairs of simultaneous equations by using elimination method.

$5x + 3y = 11$

$3x + 2y = 6$

$5x + 3y = 11$ —— 1

$3x + 2y = 6$ —— 2

1 − 2

$3x + 2y = 6$ → multiply with 3

$-\ 5x + 3y = 11$ → multiply with 2

$9x + 6y = 18$

$-10x + 6y = 22$

$-x = -4$

$x = 4$

x into 1

$3(4) + 2y = 6$

$12 + 2y = 6$ → $12 + 2y = 6$

$2y = 6 - 12$

$2y = -6$ → $2y = -6$

$y = -3$

6. Solve the following pairs of simultaneous equations by using elimination method.

$3x + 4y = 11$

$7x - y = 5$

$3x + 4y = 11$ —— 1

$7x + 4y = 5$ —— 2

1 − 2

$3x + 4y = 11$ → multiply with 7

$-\ 7x - y = 5$ → multiply with 3

$21x + 28y = 77$

$-\ 21x - 3y = 15$

$31y = 62$ → $31\ y = 62$

$y = \dfrac{62}{31}$

$y = 2$

y into 1

$3x + 4(2) = 11$

$3x + 8 = 11$ → $3\ x + 8 = 11$

$3x = 11 - 8$

$3x = 3$ → $3\ x = 3$

$x = 1$

EXERCISE 3

Solve the following pairs of simultaneous equations.

1. $2p - 5q = 11$

   $2p - 3q = 7$

   *use substitution method

2. $3x - 4y = 25$

   $4x - 5y = 32$

   *use elimination method

3. $7x - 3y = 6$

   $7x - 4y = 8$

   *use elimination method

4. $3x + 5y = 27$

   $7x - 3y = 19$

   *use substitution method

5. $x + y = \frac{1}{2}$

   $x - y = \frac{1}{4}$

   *use elimination method

6. $x + 2y = 5$

   $x + y = 1$

   *use substitution method

1.4 Algebraic Formula

EXAMPLE

1. Given $P\sqrt{\frac{N}{R}} = 3$, make N as subject in terms of P and R.

$$P\sqrt{\frac{N}{R}} = 3 \longrightarrow P\sqrt{\frac{N}{R}} = 3$$

$$\sqrt{\frac{N}{R}} = \frac{3}{P}$$

$$\frac{N}{R} = \left(\frac{3}{P}\right)^2$$

$$\frac{N}{R} = \frac{9}{P^2} \longrightarrow \frac{N}{R} = \frac{9}{P^2}$$

$$N = \frac{9}{P^2}(R)$$

$$N = \frac{9R}{P^2}$$

2. Given $\frac{p}{2x} - 1 = y$, express x in terms of p and y

$$\frac{p}{2x} - 1 = y \longrightarrow \frac{p}{2x} - 1 = y$$

$$\frac{p}{2x} = y + 1 \longrightarrow \frac{p}{2x} = y + 1$$

$$p = (2x)(y + 1) \longrightarrow p = (2x)(y + 1)$$

$$p = 2xy + 2x$$

$$p = x(2y + 2) \longrightarrow \text{factorize } x$$

$$x = \frac{p}{2y + 2}$$

$$x = \frac{p}{2(y + 1)} \longrightarrow \text{factorize } 2$$

3. $\frac{1}{p} = \frac{1}{x} + \frac{1}{y}$, make y as subject in terms of p and x.

$$\frac{1}{p} = \frac{1}{x} + \frac{1}{y}$$

$$\frac{1}{p} - \frac{1}{x} = \frac{1}{y} \longrightarrow \frac{1}{p} - \frac{1}{x} = \frac{1}{y}$$

$$\frac{1}{y} = \frac{1(x)}{p(x)} - \frac{1(p)}{x(p)}$$

$$\frac{1}{y} = \frac{x-p}{xp} \longrightarrow \frac{1}{y} = \frac{x-p}{xp}$$

$$1 = \left(\frac{x-p}{xp}\right)(y) \longrightarrow 1 = \boxed{\left(\frac{x-p}{xp}\right)}(y)$$

$$y = \frac{xp}{x-p}$$

4. $m = \frac{x}{x+y}$, make x as subject in terms of m and y.

$$m = \frac{x}{x+y} \longrightarrow m = \frac{x}{\boxed{x+y}}$$

$$m(x+y) = x$$

$$mx + my = x$$

$$x = mx + my \longrightarrow x = mx + my$$

$$x - mx = my$$

$$x(1-m) = my \longrightarrow \text{factorize } x$$

$$x = \frac{my}{1-m}$$

5. $\frac{2x+y}{2x-y} = \frac{3}{4}$, make x as subject in term of y.

$$\frac{2x+y}{2x-y} = \frac{3}{4}$$

$$(2x+y)(4) = (3)(2x-y)$$

$$8x + 4y = 6x - 3y \longrightarrow 8x + 4y = 6x - 3y$$

$$8x - 6x = -3y - 4y$$

$$2x = -7y \longrightarrow 2x = -7y$$

$$x = -\frac{7}{2}y$$

EXERCISE 4

1. Given $x = \frac{y}{y-2}$, express y in term of x.

2. Given $T = \sqrt{\frac{32}{p+1}}$, express p in term of T.

3. $V = \frac{\pi d^2 h}{4}$, express d in terms of V, π and h.

4. $p = \frac{2}{q} + \frac{1}{2q}$, make q as the subject in term of p.

5. Given $(x - y)^3 = 8$,

   (a) make x as the subject in term of y.

   (b) make y as the subject in term of x.

6. Given $y + \frac{1}{x} = 5$,

   (a) calculate the value of y if $x = \frac{1}{2}$.

   (b) express x in term of y.

7. Given $2q - p = \frac{p}{r}$, express p in terms of q and r.

1.5 Review Answer

ANSWER EXERCISE 1

1. $\frac{x^2+x-2}{x^2+4x+3} \times \frac{x^2+2x+1}{x^2-1}$

$$= \boxed{\frac{(x-1)(x+2)}{(x+1)(x+3)} \times \frac{(x+1)(x+1)}{(x+1)(x-1)}} \longrightarrow \text{Factorize the equation}$$

$$= \frac{\cancel{(x-1)}(x+2)}{\cancel{(x+1)}(x+3)} \times \frac{\cancel{(x+1)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-1)}}$$

$$= \frac{(x+2)}{(x+3)}$$

2. $\frac{2}{x-2} - \frac{4}{x(x-2)}$

$$= \frac{2(x)}{(x-2)(x)} - \frac{4}{x(x-2)}$$

$$= \frac{2x-4}{x(x-2)} \longrightarrow \text{factorize 2}$$

$$= \frac{2(x-2)}{x(x-2)} \longrightarrow \frac{2\cancel{(x-2)}}{x\cancel{(x-2)}}$$

$$= \frac{2}{x}$$

3. (a) $\frac{x^2-4}{6} \times \frac{3y}{2x+4}$

$$= \frac{\boxed{(x+2)(x-2)}}{6} \times \frac{3y}{2(x+2)} \longrightarrow \text{Factorize the equation}$$

$$= \frac{\cancel{(x+2)}(x-2)}{\cancel{6}(2)} \times \frac{\cancel{3}y}{2\cancel{(x+2)}}$$

$$= \frac{(x-2)(y)}{2(2)}$$

$$= \frac{(x-2)y}{4}$$

(b) $\frac{2x+4}{3x-6} \div \frac{x^2-4}{x^2+x-6}$

$= \frac{2(x+2)}{3(x-2)} \div \frac{(x+2)(x-2)}{(x-2)(x+3)}$ ⟶ Factorize the equation

$= \frac{2\cancel{(x+2)}}{3\cancel{(x-2)}} \times \frac{\cancel{(x-2)}(x+3)}{\cancel{(x+2)}(x-2)}$

$= \frac{2x+6}{3x-6}$ or $= \frac{2(x+3)}{3(x-2)}$

4. $\frac{1}{3x} + \frac{x-6}{12x^2}$

$= \frac{1(4x)}{3x(4x)} + \frac{x-6}{12x^2}$

$= \frac{4x+x-6}{12x^2}$

$= \frac{5x+6}{12x^2}$

5. $\frac{1}{x(x+1)} - \frac{x}{(x+1)}$

$= \frac{1}{x(x+1)} - \frac{x(x)}{(x+1)(x)}$

$= \frac{1-x^2}{x(x+1)}$ ⟶ factorize

$= \frac{(1+x)(1-x)}{x(x+1)}$ ⟶ $\frac{\cancel{(1+x)}(1-x)}{x\cancel{(x+1)}}$

$= \frac{1-x}{x}$

6. $\frac{8xy+3}{15xy^2} + \frac{1}{3y}$

$= \frac{8xy+3}{15xy^2} + \frac{1(5xy)}{3y(5xy)}$

$= \frac{8xy+5xy+3}{15xy^2}$

$= \frac{13xy+3}{15xy^2}$

7. $\frac{4x-8}{6} \div \frac{x-2}{3}$

$$= \frac{4(x-2)}{6} \div \frac{x-2}{3}$$

$$= \frac{4(x-2)}{6} \times \frac{3}{x-2}$$

$$= \frac{4\cancel{(x-2)}}{\cancel{6}(2)} \times \frac{\cancel{3}}{\cancel{x-2}}$$ Factorize the equation

$$= \frac{4}{2}$$

$$= 2$$

ANSWER EXERCISE 2

1. (a) $\frac{2x}{7} = 1$

$2x = 1(7)$

$2x = 7$

$x = \frac{7}{2}$

(b) $3p - 2 = 2(p + 3)$

$3p - 2 = 2p + 6 \longrightarrow 3p - 2 = 2p + 6$

$3p - 2p = 6 + 2$

$p = 8$

2. (a) $3p = 1 - 2p$

$3p + 2p = 1$

$5p = 1 \longrightarrow 5p = 1$

$p = \frac{1}{5}$

(b) $\frac{x + 2}{3} = x - 6$

$x + 2 = (3)(x - 6)$

$x + 2 = 3x - 18 \longrightarrow x + 2 = 3x - 18$

$x - 3x = -18 - 2$

$-2x = -20 \longrightarrow -2x = -20$

$x = 10$

3. (a) $2 - 5x = 4(1 - x)$

$2 - 5x = 4 - 4x \longrightarrow 2 - 5x = 4 - 4x$

$-5x + 4x = 4 - 2$

$-x = 2 \longrightarrow -x = 2$

$x = -2$

(b) $\frac{1}{3}(x-1)=2$

$\frac{1}{3}x-\frac{1}{3}=2 \longrightarrow \frac{1}{3}x-\frac{1}{3}=2$

$\frac{1}{3}x=2+\frac{1}{3}$

$\frac{1}{3}x=\frac{7}{3} \longrightarrow \frac{1}{3}x=\frac{7}{3}$

$x=\frac{7}{\cancel{3}}\cancel{(3)}$

$x=7$

4. (a) $\frac{2}{m}=6$

$2=6m \longrightarrow 2=6\,m$

$m=\frac{2}{6} \longrightarrow m=\frac{\cancel{2}}{\cancel{6}}$, change to simplest form

$m=\frac{1}{3}$

(b) $9-(2p+5)=4p$

$9-2p-5=4p \longrightarrow 9-2p-5=4p$

$9-5=4p+2p$

$4=6p \longrightarrow 4=6\,p$

$p=\frac{4}{6} \longrightarrow$ change to simplest form

$p=\frac{2}{3}$

5. (a) $\frac{1}{3}(x+1)=3$

$\frac{1}{3}x+\frac{1}{3}=3 \longrightarrow \frac{1}{3}x+\frac{1}{3}=3$

$\frac{1}{3}x=3-\frac{1}{3}$

$\frac{1}{3}x=\frac{8}{3} \longrightarrow \frac{1}{3}x=\frac{8}{3}$

$x=8$

(b) $2(y-5)-(y-3)=2$

$2y-10-y+3=2$

$2y-y=2+10-3$ $\longrightarrow$ $2y-10-y+3=2$

$y=9$

6. (a) $2p-\frac{1}{4}=2\frac{3}{4}$

$2p-\frac{1}{4}=\frac{11}{4}$ $\longrightarrow$ $2p-\frac{1}{4}=\frac{11}{4}$

$2p=\frac{11}{4}+\frac{1}{4}$

$2p=\frac{12}{4}$

$2p=3$ $\longrightarrow$ $2p=3$

$p=\frac{3}{2}$

(b) $2x+1=3-2(2x-1)$

$2x+1=3-4x+2$ $\longrightarrow$ $2x+1=3-4x+2$

$2x+4x=3+2+1$

$6x=6$ $\longrightarrow$ $6x=6$

$x=1$

7. (a) $\frac{x-1}{2}=5$

$x-1=(5)(2)$

$x-1=10$ $\longrightarrow$ $x-1=10$

$x=10+1$

$x=11$

(b) $3(y-4)=8-y$

$3y-12=8-y$ ⟶ $3y-12=8-y$

$3y+y=8+12$

$4y=20$ ⟶ $4\ y=20$

$y=\dfrac{20}{4}$

$y=5$

ANSWER EXERCISE 3

1. $2p - 5q = 11$

   $2p - 3q = 7$

   $2p - 5q = 11$ —— 1

   $2p - 3q = 7$ —— 2

   $2p = 11 + 5q$ —— 3

   3 into 2

   $(11 + 5q) - 3q = 7 \rightarrow 11 + 5q - 3q = 7$

   $5q - 3q = 7 - 11$

   $2q = -4 \rightarrow 2q = -4$

   $q = -2$

   q into 3

   $2p = 11 + 5(-2)$

   $2p = 11 - 10 \rightarrow 2p = 1$

   $2p = 1$

   $p = \frac{1}{2}$

2. $3x - 4y = 25$

$4x - 5y = 32$

$3x - 4y = 25$ —— 1

$4x - 5y = 32$ —— 2

1 − 2

$3x + 4y = 25$ → multiply with 4

$-\ 4x - 5y = 32$ → multiply with 3

$12x + 16y = 100$

$-\ 12x - 15y = 96$

$-y = 4$

$y = -4$

y into 1

$3x - 4(-4) = 25$

$3x + 16 = 25$ → $3x + 16 = 25$

$3x = 25 - 16$

$3\ x = 9$ → $3\ x = 9$

$x = \frac{9}{3}$

$x = 3$

3. $7x - 3y = 6$

$7x - 4y = 8$

$7x - 3y = 6$ —— 1

$7x - 4y = 8$ —— 2

1 − 2

$7x - 3y = 6$

$-\ 7x - 4y = 8$

$y = -2$

y into 2

$7x - 4(-2) = 8$

$7x + 8 = 8$ → $7x + 8 = 8$

$7x = 8 - 8$

$x = 0$

4. $3x + 5y = 27$

$7x - 3y = 19$

$3x + 5y = 27$

$7x - 3y = 19$ —— ①

$3x = 27 - 5y$ —— ②

$x = \frac{27}{3} - \frac{5}{3}y$

$x = 9 - \frac{5}{3}y$ —— ③

3 into 1

$7\left(9 - \frac{5}{3}y\right) - 3y = 19 \longrightarrow 7\left(9 - \frac{5}{3}y\right) - 3y = 19$

$63 - \frac{35}{3}y - 3y = 19 \longrightarrow 63 - \frac{35}{3}y - 3y = 19$

$-\frac{35}{3}y - 3y = 19 - 63$

$-\frac{44}{3}y = -44 \longrightarrow \frac{44}{3}y = -44$

$-44y = -44(3)$

$-44y = -132 \longrightarrow -44y = -132$

$y = 3$

y into 3

$x = 9 - \frac{5}{3}(3) \longrightarrow \cancel{x} = 9 - \frac{5}{\cancel{3}}\cancel{(3)}$

$x = 9 - 5$

$x = 4$

5. $x + y = \frac{1}{2}$

$x - y = \frac{1}{4}$

$x + y = \frac{1}{2}$ —— 1

$x - y = \frac{1}{4}$ —— 2

2 − 1

$$x + y = \frac{1}{2}$$

$$- \quad x - y = \frac{1}{4}$$

$$2y = \frac{1}{4} \longrightarrow 2y = \frac{1}{4}$$

$$y = \frac{1}{8}$$

y into 2

$$x - \left(\frac{1}{8}\right) = \frac{1}{4}$$

$$x = \frac{1(2)}{4(2)} + \frac{1}{8}$$

$$x = \frac{3}{8}$$

6. $x + 2y = 5$

$x + y = 1$

$x + 2y = 5$ ——— 1

$x + y = 1$ ——— 2

$x = 1 - y$ ——— 3

3 into 1

$(1 - y) + 2y = 5$

$1 - y + 2y = 5 \longrightarrow 1 - y + 2y = 5$

$-y + 2y = 5 - 1$

$y = 4$

y into 3

$x = 1 - (4)$

$x = -3$

ANSWER EXERCISE 4

1. $x = \frac{y}{y-2}$

$$x = \frac{y}{y-2} \longrightarrow x = \frac{y}{y\boxed{-2}}$$

$$(x)(y-2) = y \longrightarrow (x)(y-2) = y$$

$$xy - 2x = y$$

$$xy - y = 2x \longrightarrow xy - y = 2x \text{ ,factorize } y$$

$$y(x-1) = 2x \longrightarrow y(x\boxed{-1}) = 2x$$

$$y = \frac{2x}{(x-1)}$$

2. $T = \sqrt{\frac{32}{p+1}}$

$$T = \sqrt{\frac{32}{p+1}}$$

$$T^2 = \frac{32}{p+1} \longrightarrow T^2 = \frac{32}{p\boxed{+1}}$$

$$T^2(p+1) = 32 \longrightarrow T^2(p+1) = 32$$

$$p + 1 = \frac{32}{T^2} \longrightarrow p + 1 = \frac{32}{T^2}$$

$$p = \frac{32}{T^2} - 1$$

3. $V = \frac{\pi d^2 h}{4}$

$$V = \frac{\pi d^2 h}{4} \longrightarrow V = \frac{\pi d^2 h}{4}$$

$$4(V) = d^2\ \pi h \longrightarrow 4V = d^2\,\pi h$$

$$d^2 = \frac{4V}{\pi h} \longrightarrow d^2 = \frac{4V}{\pi h}$$

$$d = \sqrt{\frac{4V}{\pi h}}$$

4. $p = \frac{2}{q} + \frac{1}{2q}$

$$p = \frac{2}{q} + \frac{1}{2q}$$

$$p = \frac{2(2)}{q(2)} + \frac{1}{2q}$$

$$p = \frac{4+1}{2q}$$

$$p = \frac{5}{2q} \longrightarrow p = \frac{5}{2q}$$

$$p(2q) = 5$$

$$2pq = 5 \longrightarrow 2p\ q = 5$$

$$q = \frac{5}{2p}$$

5. (a) $(x-y)^3 = 8$

$$(x-y)^3 = 8 \longrightarrow (x-y)^3 = 8$$

$$x - y = \sqrt[3]{8}$$

$$x - y = 2 \longrightarrow x - y = 2$$

$$x = 2 + y$$

(b) $(x-y)^3 = 8$

$$(x-y)^3 = 8 \longrightarrow (x-y)^3 = 8$$

$$x - y = \sqrt[3]{8}$$

$$x - y = 2 \longrightarrow x - y = 2$$

$$x - 2 = y$$

$$y = x - 2$$

6. (a) $y + \frac{1}{x} = 5$

$$y + \frac{1}{x} = 5 \ \ , x = \frac{1}{2}$$

$$y + \frac{1}{\frac{1}{2}} = 5$$

$$y + 2 = 5$$

$$y = 5 - 2$$

$$y = 3$$

(b) $y + \frac{1}{x} = 5$

$$y + \frac{1}{x} = 5 \longrightarrow y + \frac{1}{x} = 5$$

$$\frac{1}{x} = 5 - y \longrightarrow \frac{1}{x} = 5 - y$$

$$1 = (x)(5 - y) \longrightarrow 1 = (x)\boxed{(5 - y)}$$

$$x = \frac{1}{(5 - y)}$$

7. $2q - p = \frac{p}{r}$

$$2q - p = \frac{p}{r} \longrightarrow 2q - p = \frac{p}{r}$$

$$(r)(2q - p) = p \longrightarrow (r)(2q - p) = p$$

$$2qr - pr = p \longrightarrow 2qr - pr = p$$

$$2qr = p + pr \longrightarrow 2qr = \boxed{p + pr} \quad \text{,factorize } p$$

$$2qr = (p)(1 + r)$$

$$(p)(1 + r) = 2qr \longrightarrow (p)\boxed{(1 + r)} = 2qr$$

$$p = \frac{2qr}{(1 + r)}$$

CHAPTER 2

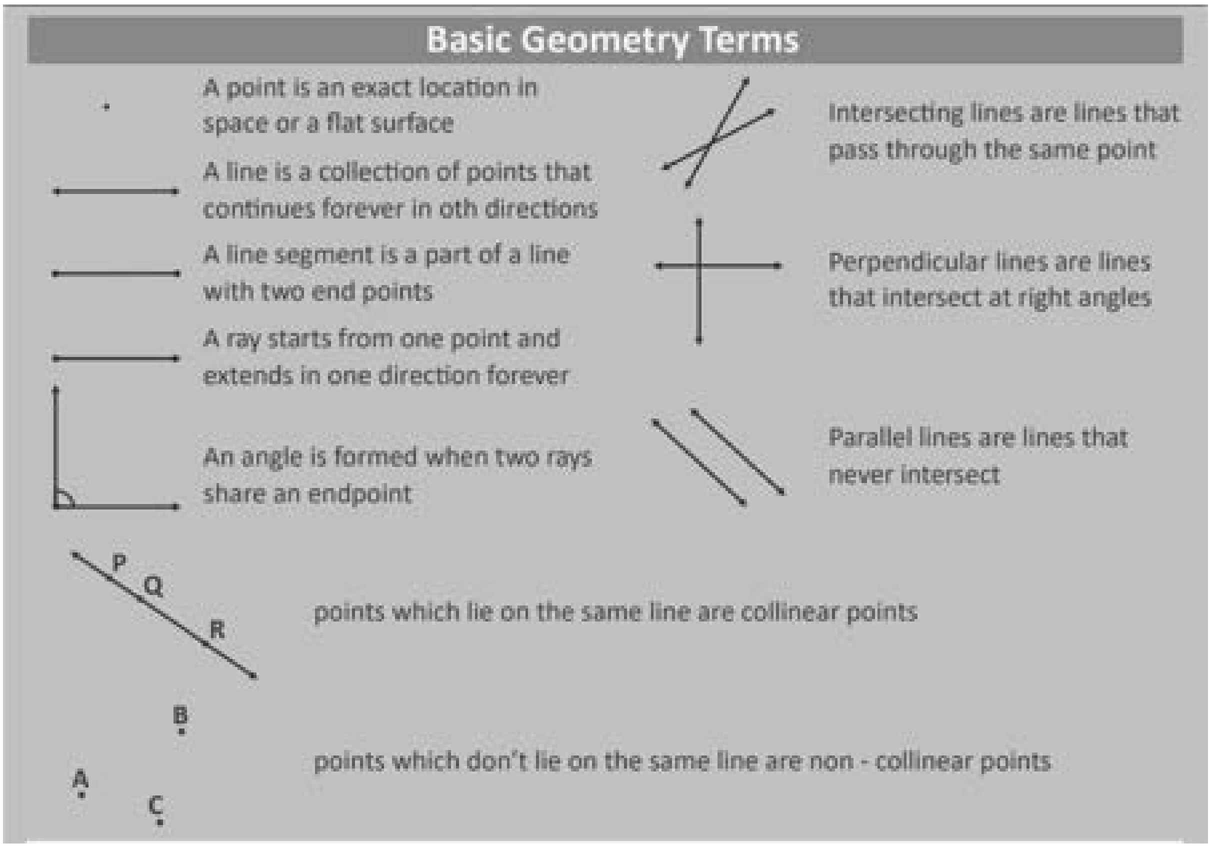
GEOMETRY

Written By: En. Mohd Shakirurahman Bin Ismail

GEOMETRY

2.1 Basic of Geometry

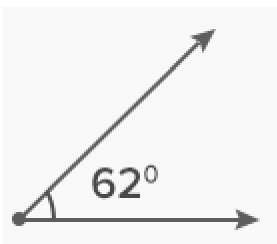
Do you want to know what Geometry is? This word is actually derived from the Greek word ' geometron' is actually made of two words – Geo and Metron. So, geometry is the mathematical study of shapes and figures. It can be seen everywhere in our everyday life. Let us study this topic in detail



2.2 Type of Angle

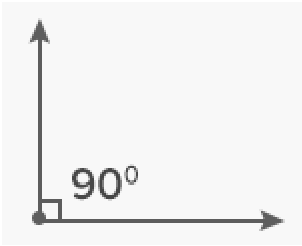
Acute angle

An acute angle is an angle that measures between 90° and 0° meaning it is smaller than a right angle. Angles is measured in degrees (°).



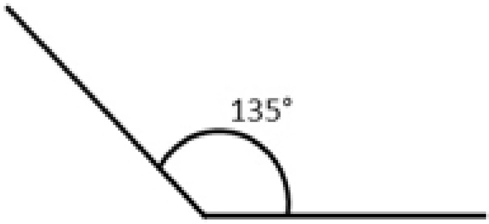
Right angle

A right angle is an angle of exactly 90°, corresponding to a quarter turn



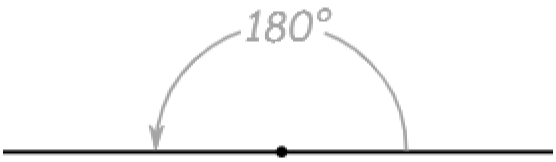
Obtuse angle

An obtuse angle is an angle that measures more than 90° and less than 180° degrees



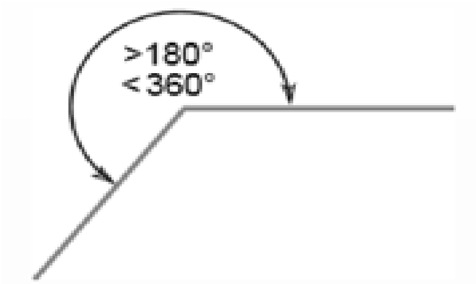
Straight angle

An angle that is exactly 180°. A straight angles changes the direction to point the opposite way



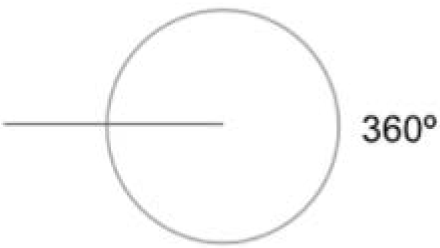
Reflect angle

An angle whose measure is greater than 180° but less than 360°



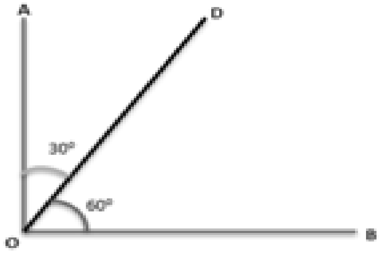
Full angle

An angle that is exactly 360°



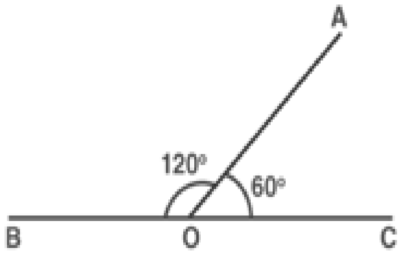
Complimentary angles.

Angles that add up to 90°



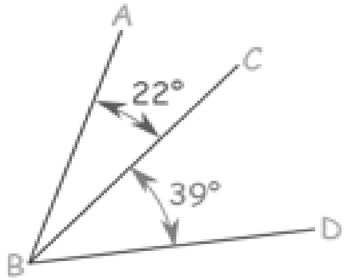
Supplementary angles

Supplementary angles are two angles whose measures add up to 180°.



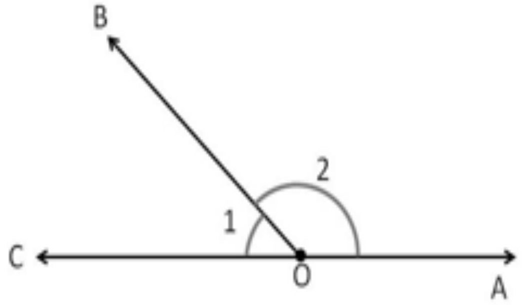
Adjacent angles

Adjacent angles are two angles that have a common side and a common vertex (corner point)



Linear pair of angles

Adjacent pair angles that form a straight line, can be defined as two adjacent that add up to 180°.

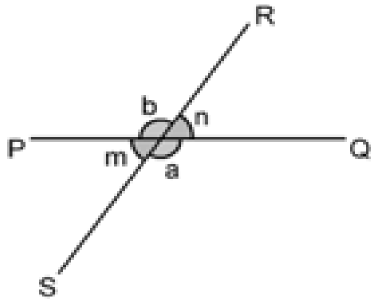


Vertically opposite angle

Angle that have a common vertex and whose arms are formed by the same lines.
Vertically opposite angles are equal

$\angle m = \angle n$

$\angle a = \angle b$



Example 2.1

Determine the name of angle below.

i) 76°

ii) 114°

iii) 90°

iv) 181°

v) 360°

vi) 180°

Solution:

i) Acute angle

ii) Obtuse angle

iii) Right angle

iv) Reflex angle

v) Full angle

vi) Straight angle

2.3 Parallel Lines and Transversal

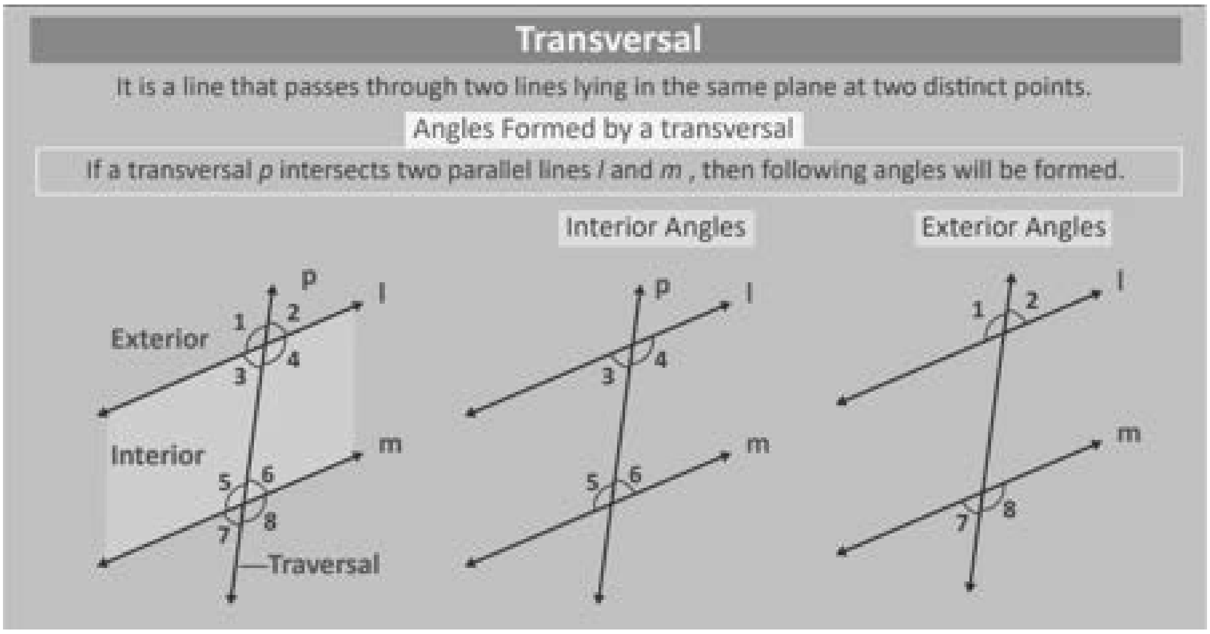
What are the lines and line segments? A line is a straight path that is endless in both directions. That means its extends in both directions without end. A line segment is a part of a line. The main difference the line and the line segment is that lines do not have endpoints while line segments have endpoints.

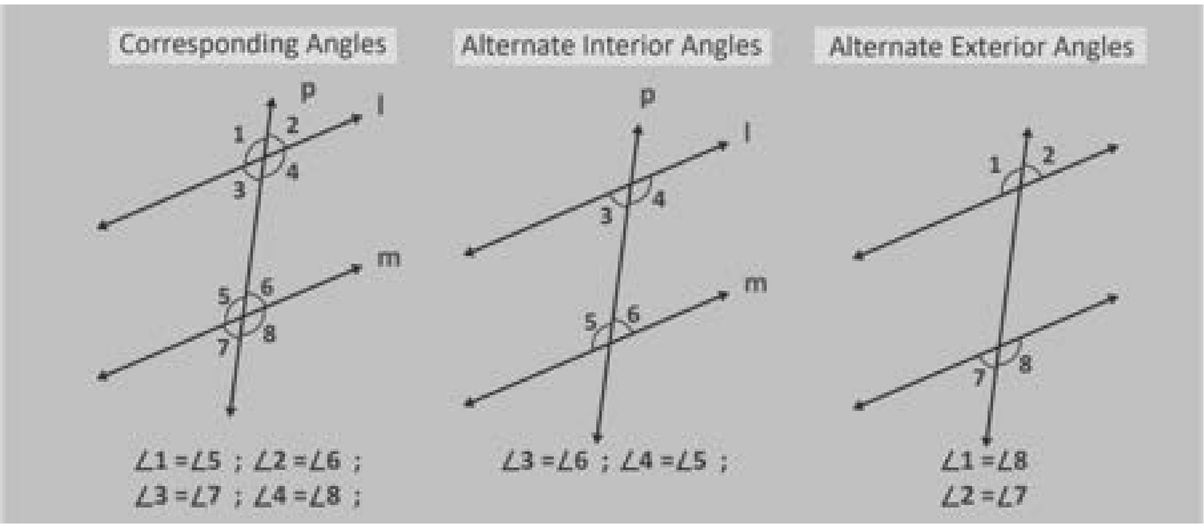
2.3.1 What are parallel lines.

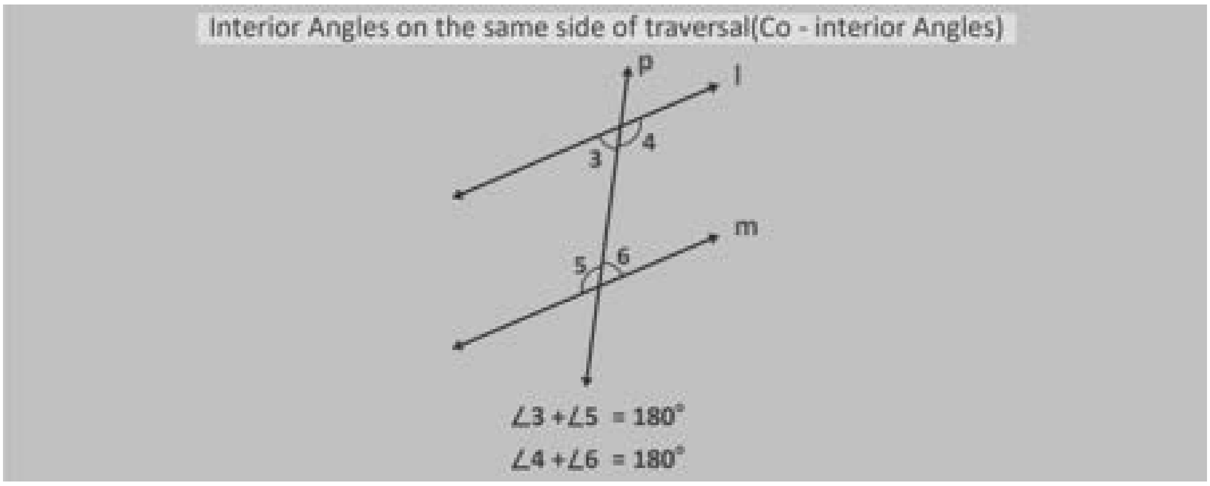
When the distance between a pair of lines is always the same. Then we call such lines parallel lines. The symbol for "parallel to" is "//". Parallel lines are the lines which never meet each other. For the two lines to be parallel, the most important things is that they are drawn in the same plan. These lines are always equidistant from each other.

2.3.2 What is a Transversal

A transversal is a line that passes through two lines lying in the same plane at two distinct points. In the transversal, the two givens lines may be parallel or non-parallel

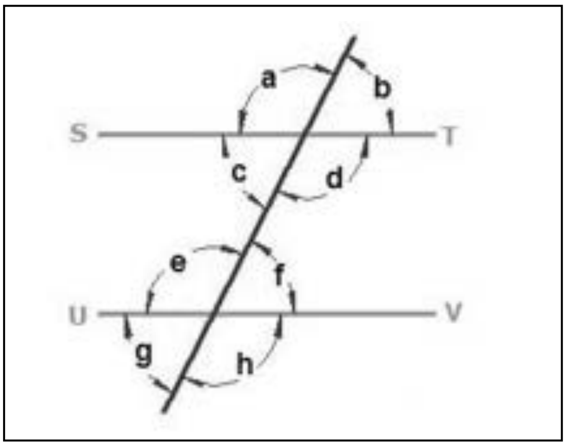






Example 2.2

ST and UV are parallel lines. c and e are:



A. Consecutive interior angles

B. Vertical angles

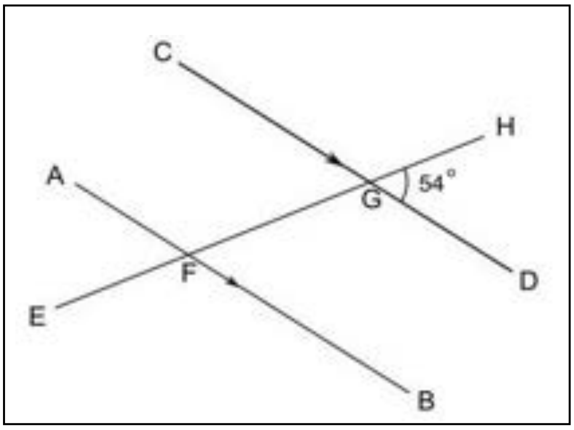
C. Alternate interior angles

D. Corresponding angles

Solutions

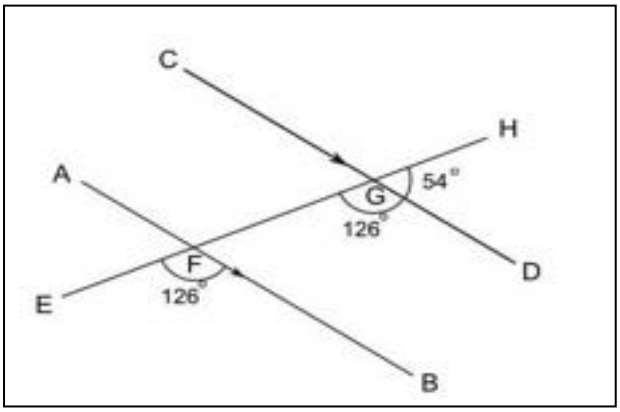
The answer is A because c and e are consecutive interior angles because they lie between the parallel lines on the same side of the transversal, and they add to 180°.

Example 2.3



AB and CD are parallel and EH is a transversal. What is the size of angle EFB?

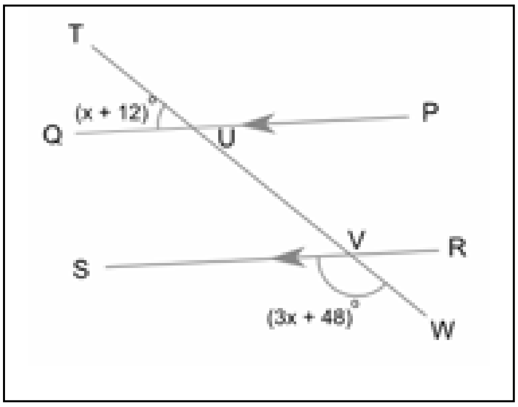
Solutions:



Angles HGD and FGD are adjacent angles on a straight line, so add to 180° so ∠FGD = 180° - 54° = 126°

Angles EFB and FGD are corresponding angles and since AB is parallel to CD. They must be equal. So, ∠126°

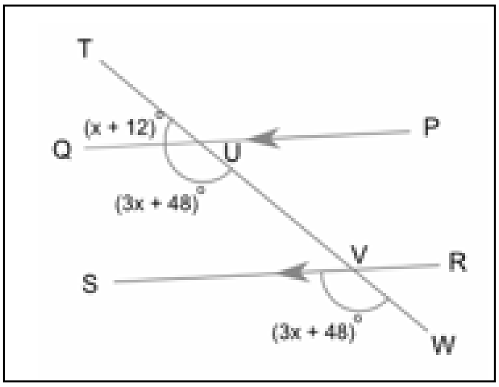
Example 2.4



PQ and RS are parallel lines and TW is a transversal. The size of angle TUQ is $(x + 12)$ ° and the size of angle SVW is $(3x + 48)$ °.

What is the value of x?

Solutions:



Angles SVW and QUV are corresponding angles, so are equal. So, $\angle QUV = (3x + 48)°$ also angles QUV and TUQ are adjacent angles on a straight lines, so add to 180°:

$(x + 12)° + (3x + 48)° = 180°$

$x + 12° + 3x + 48° = 180°$

$4x + 60° = 180°$

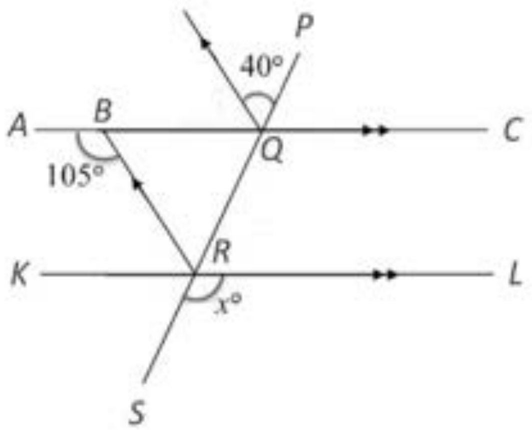
$4x = 180° - 60°$

$4x = 120°$

$x = 30°$

Example 2.5

From the diagram below, PQRS, ABQC and KRL are straight line



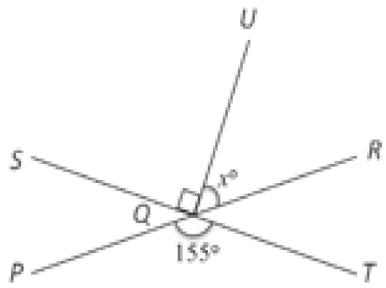
Fine the value of X

Solutions:

$\angle KRB + \angle ABR = 180°$
$\angle KRB + 105° = 180°$
$\angle KRB = 180° - 105°$
$\angle KRB = 75°$
$\angle BRQ = 40°$
$X° = \angle SRL = \angle KRQ$
$X° = 75° + 40°$
$X° = 115°$

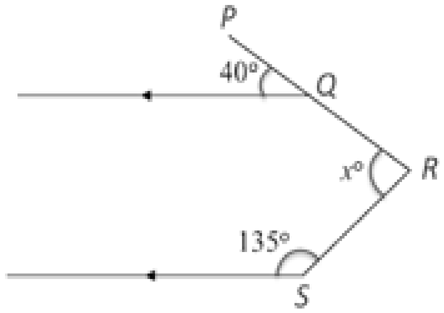
Exercise

1. In the figure below, PQR and SQT are straight lines.

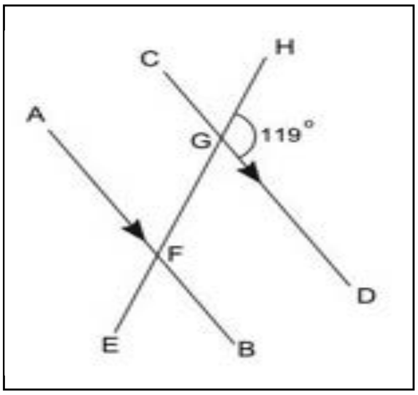


Find the value of x

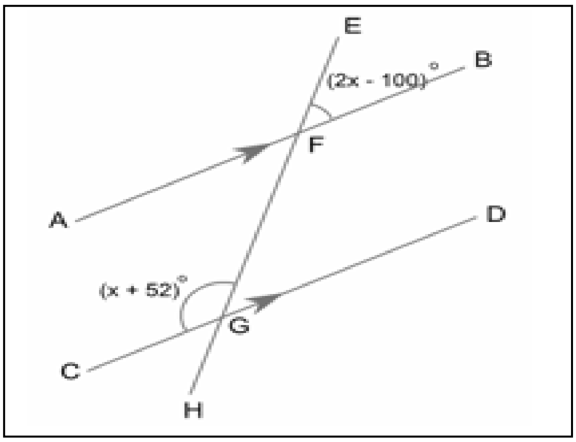
2. In the figure below, PQR is a straight line. Fine the value of x



3. AB and CD are parallel lines and EH is a transversal. What is the size of angle AFG?

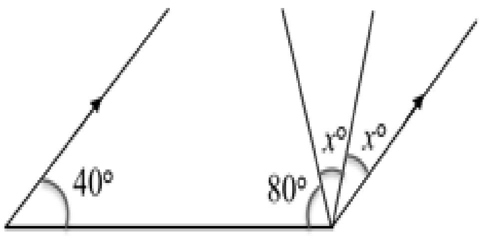


4. AB and CD are parallel lines and EH is a transversal. The size of angle EFB is (2x – 100) ° and the size of angle CGF is (x + 52) °

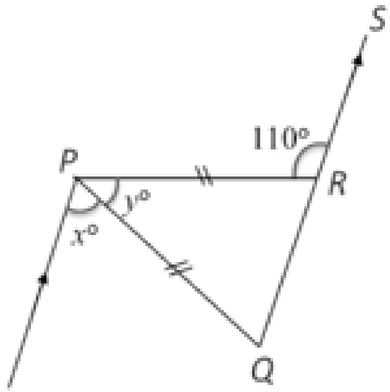


What is the actual size of the angle EFB?

5. Refer to the diagram below, fine x



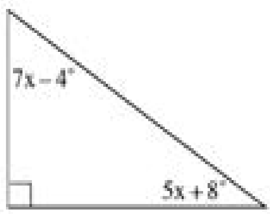
6. In the figure below, PQR is an equilateral triangle and QRS is a straight line.



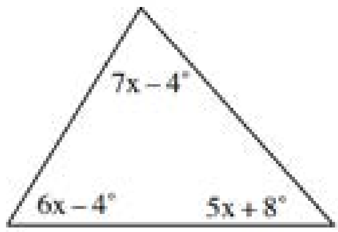
Find the values of x and y

7. Use the geometric properties and theorems you have learned to solve for x in each diagram

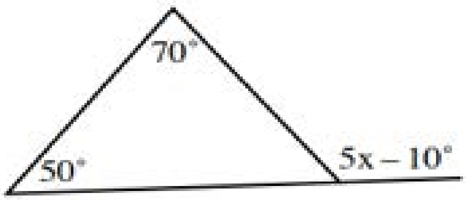
i.



Ii.



8. Find the value of x



Review Answer:

1. 65°
2. 85°
3. 61°
4. 52°
5. 30°
6. 70°
7. i. 7.167° ii. 10°
8. 26°

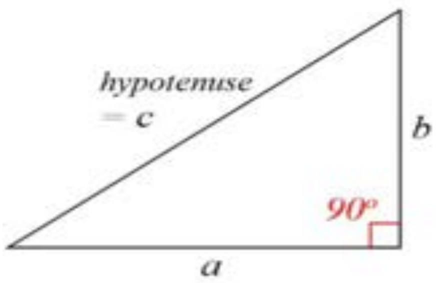
2.4 Pythagoras Theorem

In mathematics, the Pythagorean theorem, or Pythagoras theorem is a fundamental relation in Eucliden geometry among the three sides of a right triangle. It states that the area of the square whose side is the hypotenuse (the side opposite the right angle is equal to the sum of the areas of the square on the other two sides.

2.5 What is a Hypotenuse

We brief touched on what the hypotenuse is above, but simply put, the hypotenuse is the longest side of a right triangle. You can find the hypotenuse opposite from the right angle in the triangle itself, it is the only side that is not touching the 90 degrees' angle.

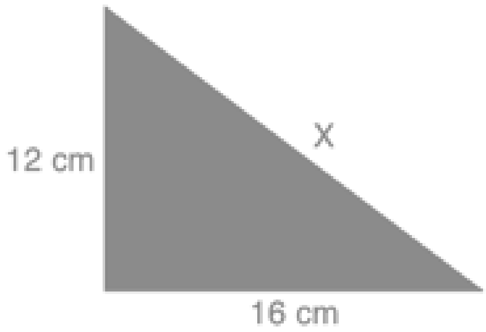
The theory of Pythagoras is one of the most relevant mathematical laws of all time thanks to the huge amount of application that can exist thanks to important theory. The theory of Pythagoras, also known as hypotenuse theory, shows us the main relations that exists within right triangle and has been famous for a very long time. This theory state that when we square the side opposite the right angle, which we call the hypotenuse, we get the same value as if we square each of the remaining sides, known as legs and add them together.



$$c^2 = a^2 + b^2$$

Example

Find x of the triangle



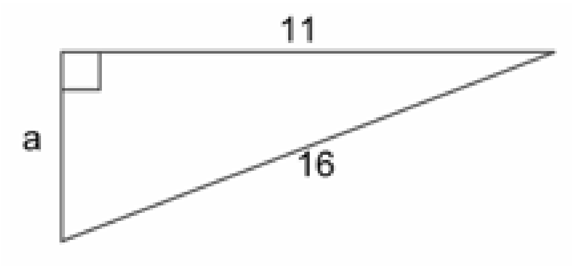
Solutions

Once again, we've got a right triangle. We're also findings the hypotenuse in this case.

Let x = c, which is our hypotenuse.

$$c^2 = a^2 + b^2$$
$$x^2 = 12^2 + 16^2$$
$$x^2 = 144 + 256$$
$$\sqrt{x^2} = \sqrt{400}$$
$$x = 20cm$$

Example



Find a.

Solutions

$A^2 + b2 = c2$

$A^2 + 11^2 = 16^2$

$A^2 + 121 = 256$

$A2 = 256 - 121$

$a = \sqrt{135}$

Example

The diagonal and one side length of triangular side 25cm and 24cm, respectively. What is the dimension of the third side?

Solutions

Using Pythagoras Theorem, $c2 = A^2 + b2$

Let b = third side

$c2 = A^2 + b2$

$25^2 = 24^2 + b^2$

$625 = 576 + b^2$

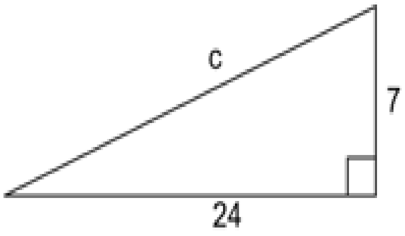
$625 - 576 = b^2$

$49 = b^2$

$7 = b$

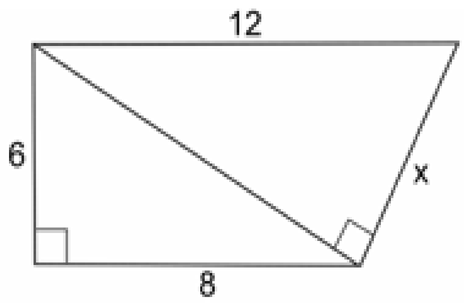
Exercise

1. Find c

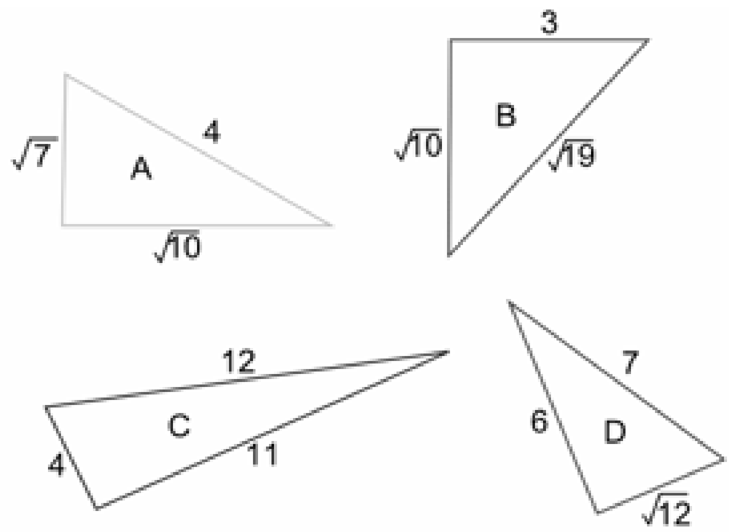


2. What is the length of the diagonal of a rectangle of length 3 and width 2?

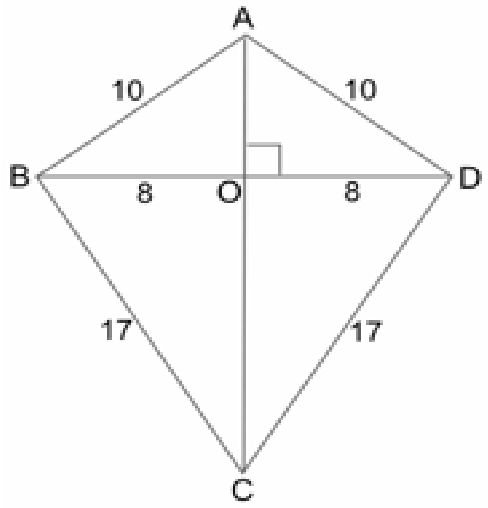
3. What is the length of the side x?



4. Only one of these triangles is really a right triangle. Which one?



5. The diagram shows a kite ABCD



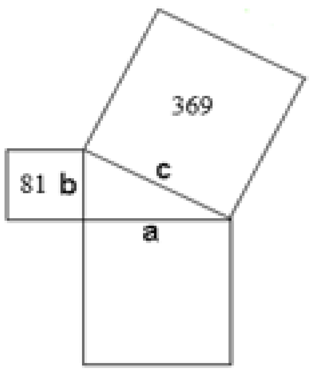
The diagonal cut at right angles and intersect at O. What is the length of the diagonal AC?

6. Town B is 8 miles north and 17 miles west of town A. How far are the two towns apart?

7. A 3m ladder stands on horizontal ground and reaches 2.8 m up a vertical wall. How far is the foot of the ladder from the base of the wall

8. A rectangular field is 125 yards long and the length of one diagonal of the field is 150 yards. What is the width of the field?

9. Find the area of the missing square off of leg b if the area of one square is 369 and the area of another is 81



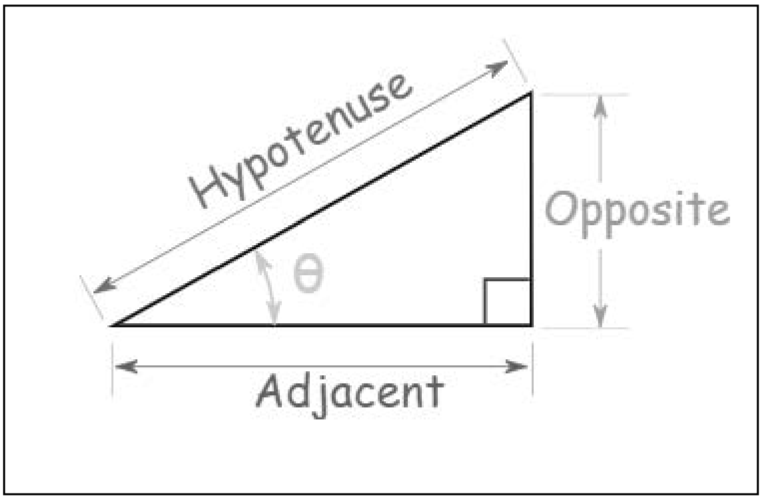
Review Answer

1. 5
2. $\sqrt{13}$
3. $\sqrt{44}$
4. B
5. 21
6. 18.8 miles
7. 1.08 m
8. 82.9 yards
9. 288

2.6 Fundamental of Trigonometric Function

Trigonometry, the branch of mathematics concerned with specific functions of angles and their applications to calculations. There are six functions of an angle commonly used in trigonometry. Their names and abbreviations are sine (sin), cosine (cos), tangent (tan), cotangent (cot), secant (sec) and cosecant (csc). These six trigonometric functions in relation to a right triangle are displayed in the figure.

The triangle of most interest is the right angled triangle. The right angle is shown by the little box in the corner:

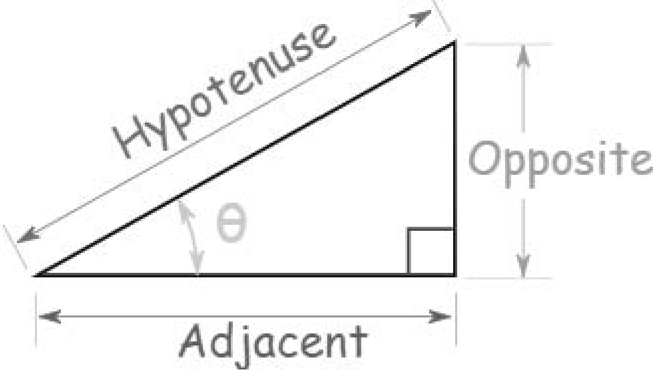


Another angle is often labeled θ, and the three sides are often called:

- Adjacent: adjacent (next to) the angle θ
- Opposite: opposite the angle θ
- And the longest side is the Hypotenuse

2.6.1 Sine, Cosine and Tangent

The main functions in trigonometry are Sine, Cosine and Tangent. They are simply one side of a right-angled divided by another. For any angle "θ":



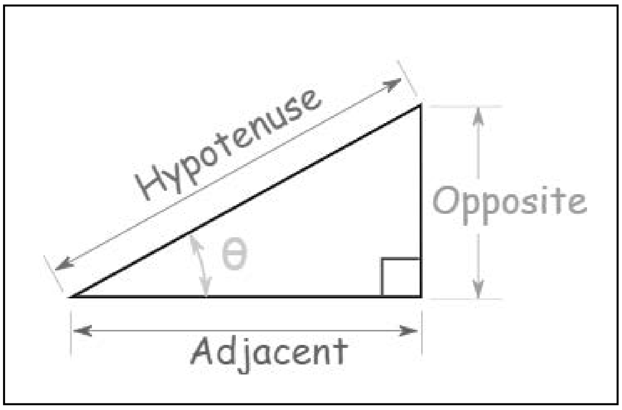
$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

2.6.2 The Reciprocal Trigonometric Ratio (Cotangent, Secant, Cosecant)

It is useful to use the reciprocal ratios frequently, depending on the question. Similar to sine, cosine and tangent, there are three other trigonometric functions which are made by dividing one side by another.

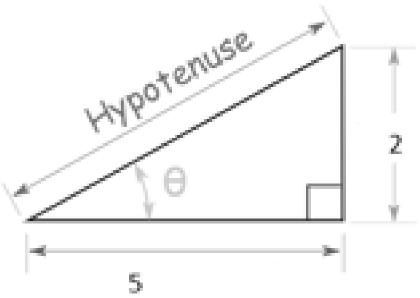


Cosecant function, Cot $\theta = \dfrac{hypotenuse}{opposite}$ **, Cot** $\theta = \dfrac{1}{\sin\theta}$

Secant Function, Sec $\theta = \dfrac{hypotenuse}{adjacent}$ **, Sec** $\theta = \dfrac{1}{\cos\theta}$

Cotangent Function, Cot $\theta = \dfrac{adjacent}{opposite}$ **, Cot** $\theta = \dfrac{1}{\tan\theta}$

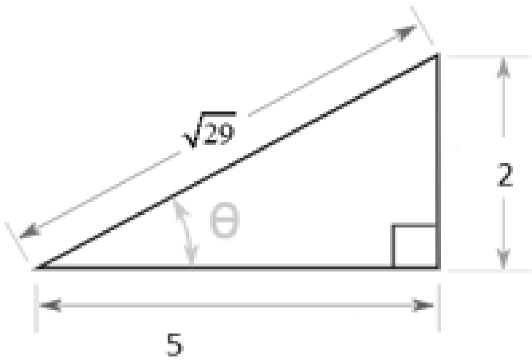
Example



Find the exact value of the six trigonometric functions for the following angles.

Solutions

Using the Pythagorean Theorem to find the length of the hypotenuse, we have that the length of the hypotenuse is $5^2 + 2^2$, thus we have that $\sqrt{29}$



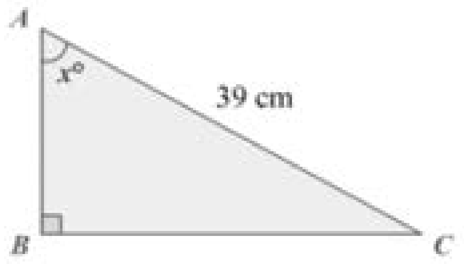
$$\cos\theta = \frac{adj}{hyp} = \frac{2}{\sqrt{29}} \qquad \sec\theta = \frac{hyp}{adj} = \frac{\sqrt{29}}{2}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{5}{\sqrt{29}} \qquad \csc\theta = \frac{hyp}{opp} = \frac{\sqrt{29}}{5}$$

$$\tan\theta = \frac{opp}{adj} = \frac{5}{2} \qquad \cot\theta = \frac{adj}{opp} = \frac{2}{5}$$

Example

In figure below, ABC is a right triangle.



Given cos x° = $\frac{5}{13}$, calculate the length of AB

Solution

$$\cos x = \frac{AB}{AC}$$

$$\cos x = \frac{5}{13}$$

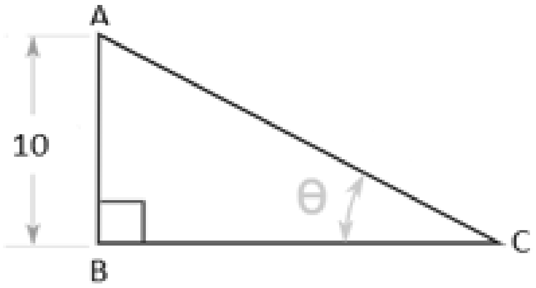
$$\cos = \frac{adj}{hyp}$$

$$\frac{AB}{39} = \frac{5}{13}$$

$$AB = \frac{5}{13}x39$$

$$= 15cm$$

Example



In figure above, ABC is a right triangle. The angle of ϴ = 25° and AB is 10cm. Calculate:

i. The length of BC

ii. The length of AC

Solution

i. The length of BC

Only tan ϴ can be used to find the length of BC.

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 25 = \frac{10cm}{BC}$$

$$BC(\tan 25) = 10cm$$

$$BC = \frac{10cm}{\tan 25}$$

$$BC = 21.45cm$$

ii. The length of AC

Only sin ϴ can be used to find length of AC

$$\sin\theta = \frac{opp}{hyp}$$

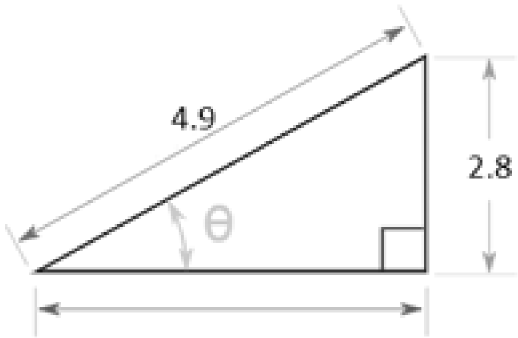
$$\sin 25 = \frac{10cm}{AC}$$

$$AC(\sin 25) = 10cm$$

$$AC = \frac{10cm}{\sin 25}$$

$$AC = 23.66cm$$

Example



The right triangle has a hypotenuse of length 4.9, an opposite side of length is 2.8. what is the value of Θ in degree?

Solution

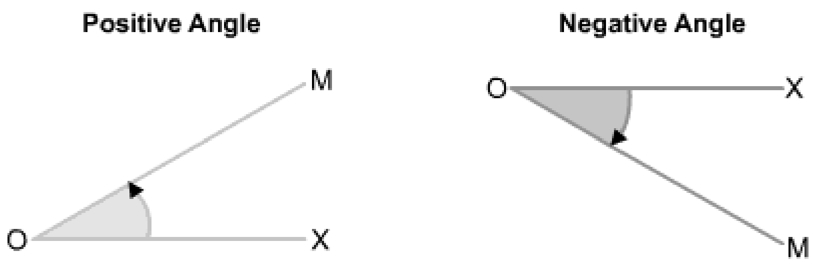
Length are only to one decimal place:

$$\sin\theta = \frac{opp}{hyp}$$
$$\sin\theta = \frac{2.8}{4.9}$$
$$\sin\theta = 0.57$$
$$\theta = \sin^{-1} 0.57$$
$$\theta = 35^0$$

2.7 Use quadrant to Determine the Value of Trigonometric Functions

On a coordinated grid a general angle is measured from the positive x-axis and is represented by the angle through which a line OM rotates about the origin.

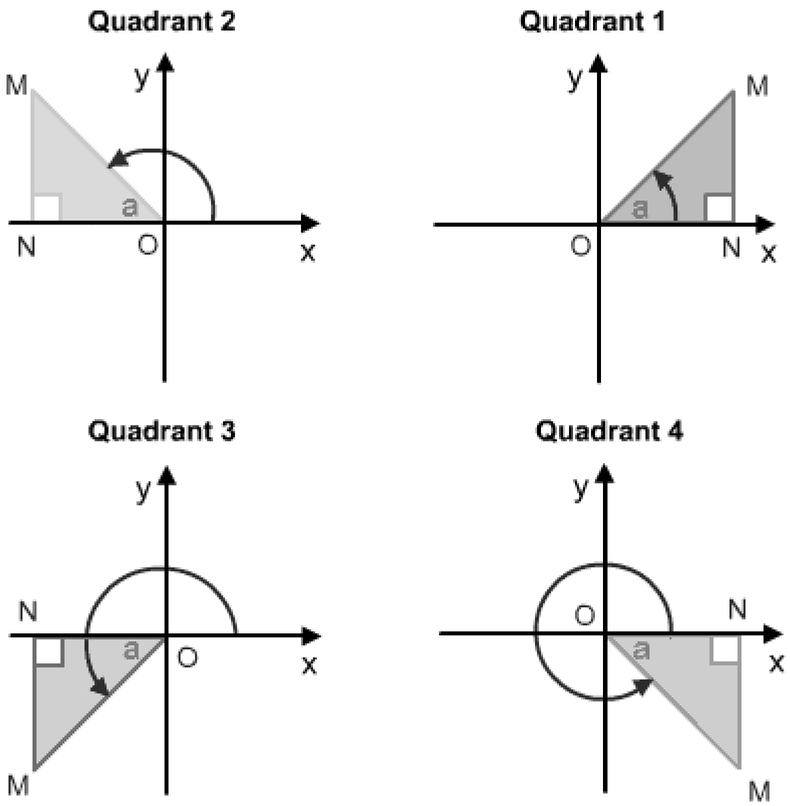
When we rotate anti-clockwise, the angle is positive while a clockwise rotation gives a negative angle.



Trigonometric functions for any angle

As the line OM rotates, the point M moves to the first quadrant where its coordinates are both positive, and into the second quadrant, where the x-coordinate becomes negative.

In the third quadrant, both coordinates are negative and finally, in the fourth quadrant, the point has a positive x- and negative y-coordinate.



First quadrant

***All the functions are positive**

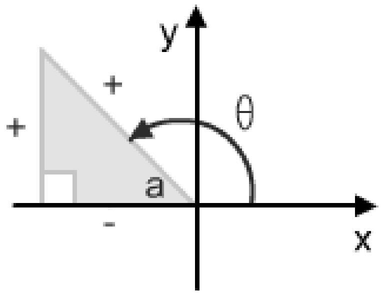
The acute angle a = θ

(Note: if OM is rotated through more than a complete revolution, then a = θ - 360°

Second quadrant

The acute angle a = 180° - θ

By looking at the signs of the coordinates of M, we see that the trigonometric functions of are:



$\sin\theta = +\sin a$

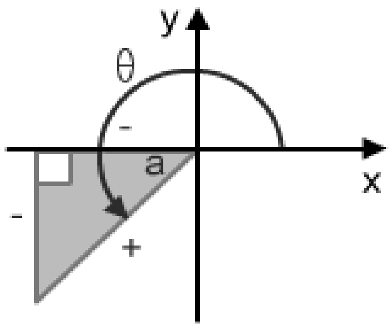
$\cos\theta = -\cos a$

$\tan\theta = -\tan a$

Third quadrant

The acute angle a = θ - 180°

The signs of the coordinates of M show us that the trigonometric functions are:



$\sin\theta = -\sin a$

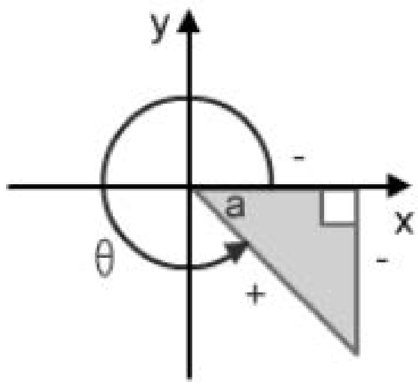
$\cos\theta = -\cos a$

$\tan\theta = +\tan a$

Fourth quadrant

The acute angle a = 360° - θ

The signs of the coordinates of M show us that the trigonometric functions of are:

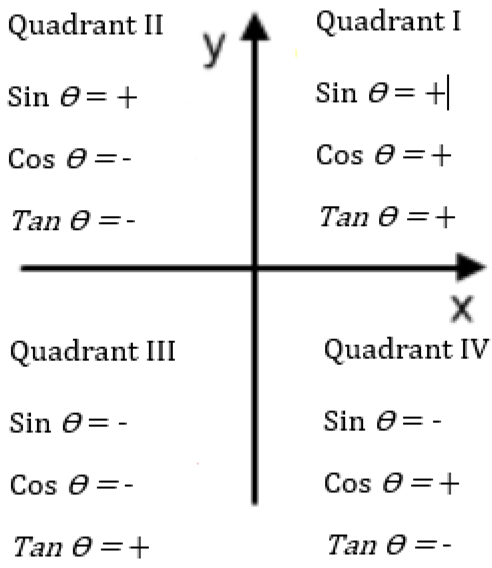


$\sin\theta = -\sin a$

$\cos\theta = +\cos a$

$\tan\theta = -\tan a$

This can be summarized as



Example

Find the value of

i. Sin 150°

ii. Sin 210°

iii. tan 300°

Solutions

i. Sin 150° = + sin (180° - 150°)

= sin 30°

= 0.5

Sin is positive in quadrant II

ii. Sin 210° = -sin (210° - 180°)

= - sin 30°

= -0.5

Sin is negative in quadrant III

iii. Tan 300° = - tan (360° - 300°)

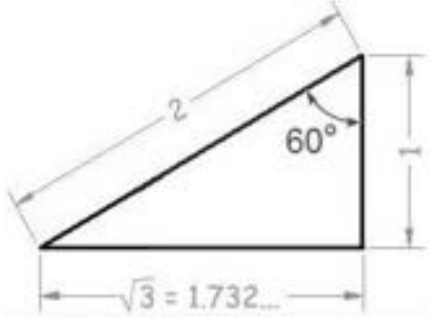
=- tan 60°

= - $\sqrt{3}$

Tan is negative in quadrant IV

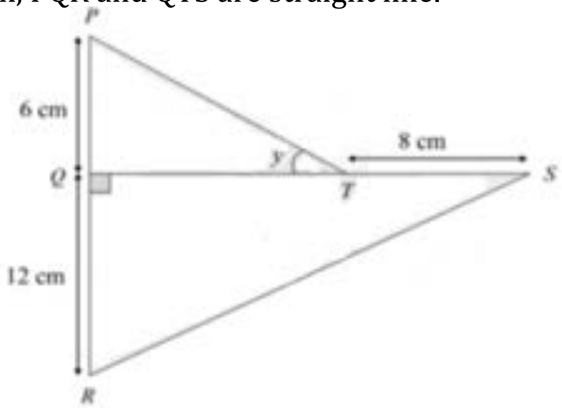
Exercise

1. The classic 60° triangle has a hypotenuse of length 2, an opposite side of length √3 and an adjacent side of length 1:



What is the value of cos 60°?

2. Based on diagram, PQR and QTS are straight line.

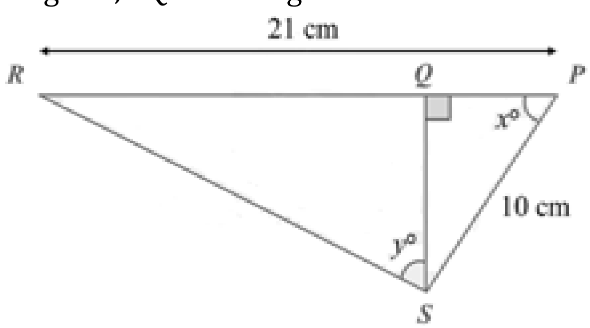


Given tan y° = ¾;

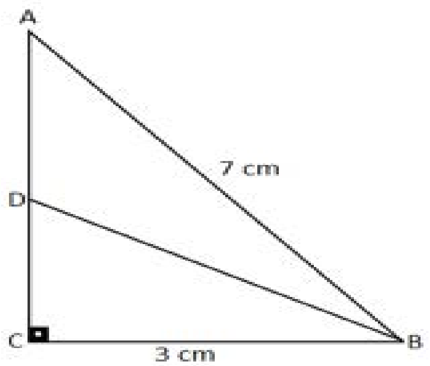
i. Find cos y°

ii. Calculate the length of RS

3. From the diagram, PQR is straight line



Given $\cos x = \frac{3}{5}$, so $\sin y = ?$

4. 

By referring to figure above, if DA = 2 cm, find:

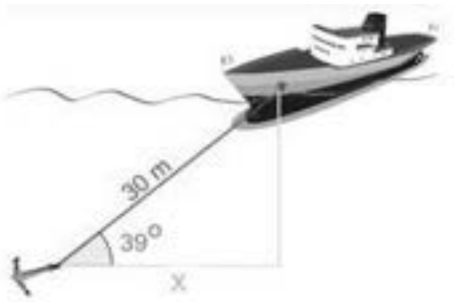
i. ∠CAB

ii. Length of CD

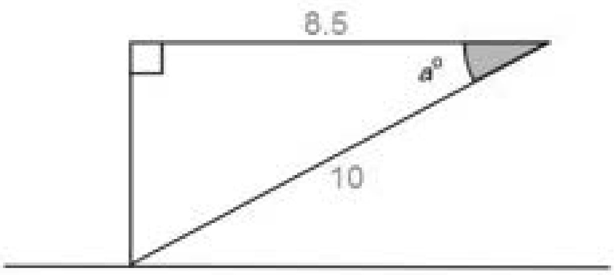
iii. ∠CBA

iv. Length of BD

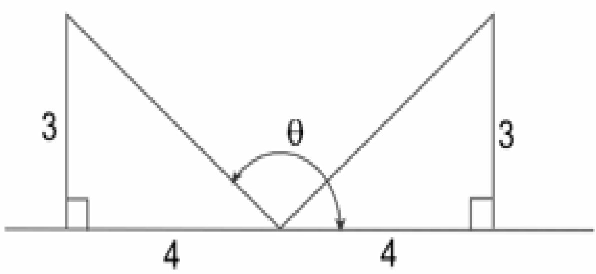
5. Use the trigonometry function to find x, the distance across the seabed



6. What is the size of angle a°?



7. What is the size of angle Θ?



8. Find the value of

   i. Cos 120°

   ii. Tan 225°

   iii. Cos -160°

   iv. Tan -30°

Review Answer

1. 0.5

2. i. $\frac{8}{10}$ ii. 20 cm

3. $\sin y = \frac{15}{17}$

4. i. 25.38° ii. 4.32 cm iii. 34.78° iv. 5.62 cm

5. 23.31 m

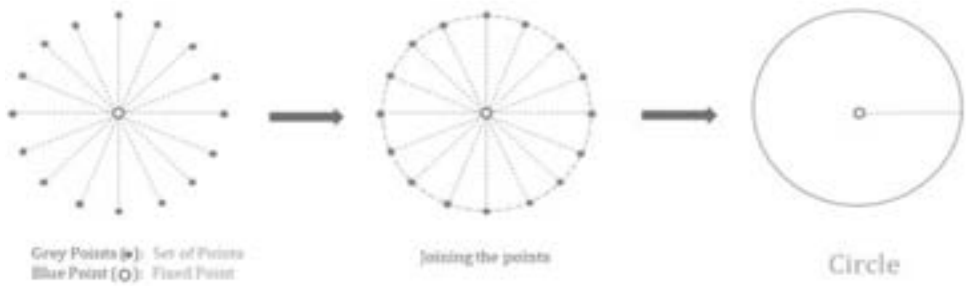
6. 31.8°

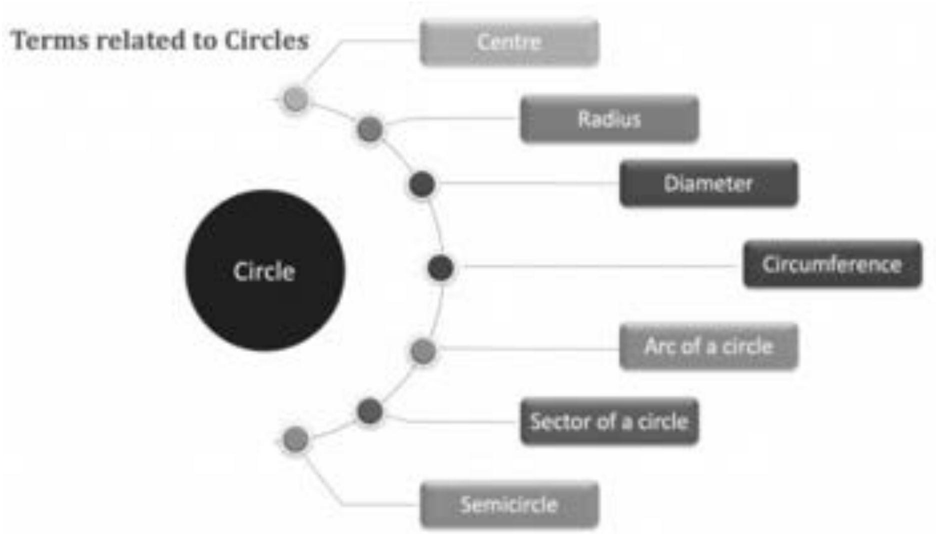
7. 143.1°

8. i. -0.5 ii. 1 iii. -0.9397 iv. -0.5773

2.8 Properties of Circle

A circle is a shape formed by tracing a point that moves in a plane such that its distance from a given point is constant. The word circle is derived from the Greek word kirkos, meaning hoop or ring. When a set of all points that are at a fixed distance from a fixed point are joined then the geometrical figure obtained is called circle.



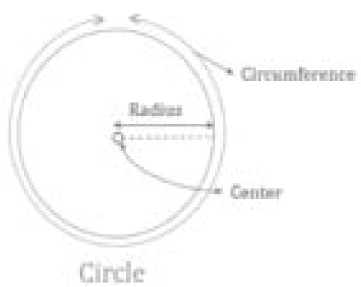


Center: The fixed point in the circle is called the center. So, the set of points are at fixed distance from the center of the circle

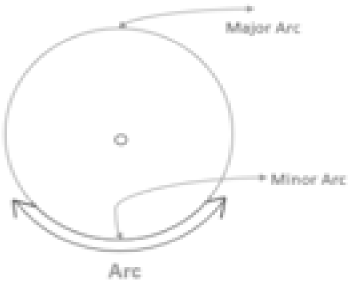
Radius: Radius is the fixed distance between the center and the set of points. It is denoted by *"R".*

Diameter: Diameter is a line segment, having boundary points of circles as the endpoints and passing through the center.

Circumference: It is the measure of the outside boundary of the circle. So, the length of the circle or perimeter of the circle is called Circumference.

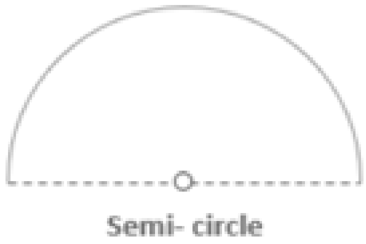


Arc of a circle: The arc of a circle is a portion of the circumference. From any points that lie on the boundary of the circle, two arcs can be created: a minor and a major arc



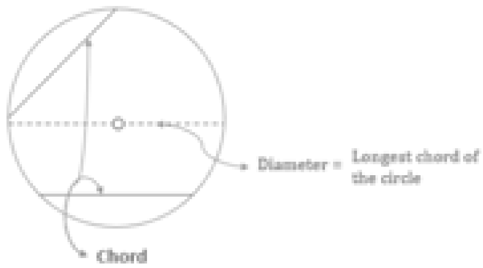
Sector of a circle: A Sector is formed by joining the endpoints of an arc with the center. On joining the endpoints with the center, two sectors will be obtained; Minor and Major

Semi-circle: A semi-circle is half part of the circle or a semi-circle is obtained when a circle is divided into two equal parts.

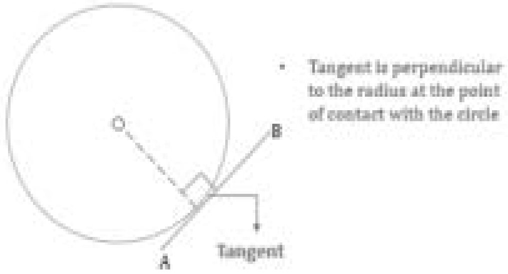


Important properties of circle – lines

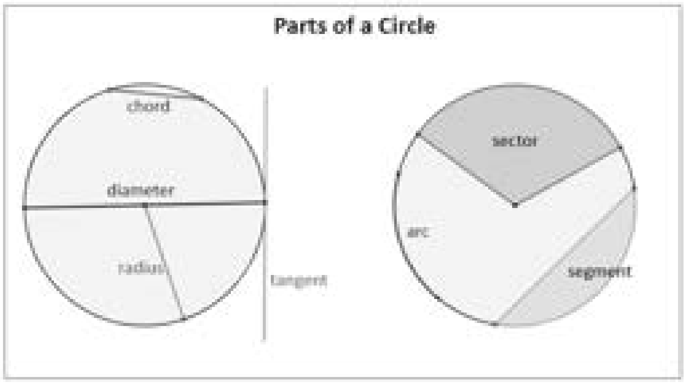
Chord: A chord is a line segment whose endpoints lie on the boundary of the circle



Tangent: Tangent is a line that ***touches*** the circle at any point

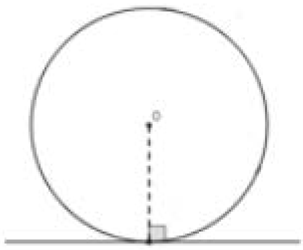


The following figures show the different part of a circle;

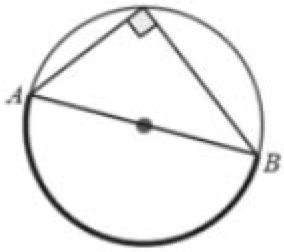


The following diagram shows some circle theorems: angle in a semicircle, angle between tangent and radius of a circle, angle at the centre of a circle is twice the angle at the circumference, angle in the same segment are equal, angles in opposite segment are supplementary; cyclic quadrilaterals and alternate segment theorem.

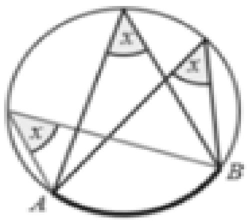
2.9 Circle Theorem



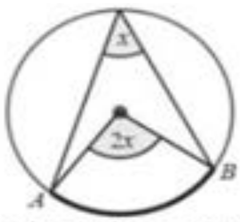
Angle between a tangent and a radius is 90°



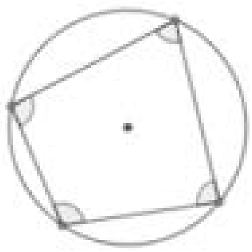
Angle in a semicircle is 90°



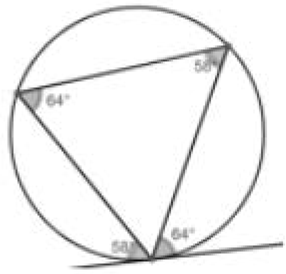
Angles in the same segment are equal



Angle at the centre of a circle is twice the angle at the circumference



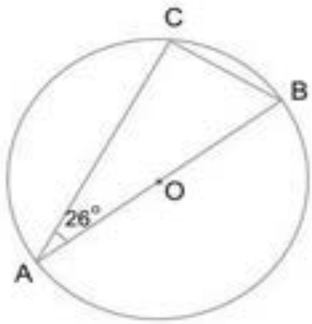
Angles in opposite segments are supplementary; cyclic quadrilaterals



Angles in alternate segment are equal

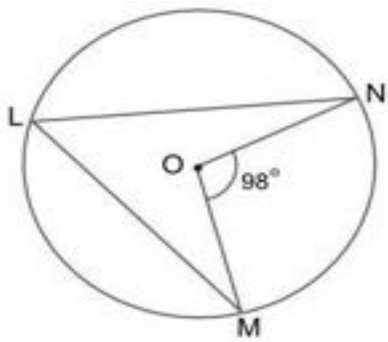
Example

AB is a diameter of a circle, center O. C is a point on the circumference of the circle, such that ∠CAB = 26°. What is the size of ∠CBA?



Solution
Since AB is a diameter of the circle, it divides the circle into two semicircles.
The Angle in the Semicircle Theorem tells us that ∠ACB = 90°
Now use angles of a triangle add to 180° to find ∠CBA:
∠CBA + 26° + 90° = 180°
⇒ ∠CBA = 64°

Example
Given L, M and N are points on the circumference of a circle, center O. ∠MON = 98°
What is the size of ∠MLN?

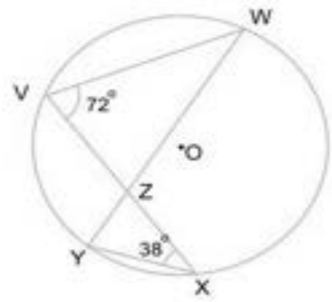


Solution

The angle at the Center Theorem tells us that
∠MLN = 2 x ∠MLN
So, ∠MLN = ½ x ∠MON
∠MLN = ½ x 98°
∠MLN = 49°

Example

V W, X and Y are points on the circumference of a circle, center O. Chords VX and WY intersect at the point Z



$\angle XVW = 72°$ and $\angle VXY = 38°$

What is the size of $\angle VZW$?

Solution

$\angle VWY$ and $\angle VXY$ are both subtended by the same arc, VY.

The Angles subtended by the Same Arc Theorem tells us that these angles are equal

$\Rightarrow \angle VWY = 38°$

This is the same angle as $\angle VWZ = 38°$

Now use angles of a triangle add to 180° in triangle VWZ

$\angle VZW + \angle WVZ + \angle VWZ = 180°$

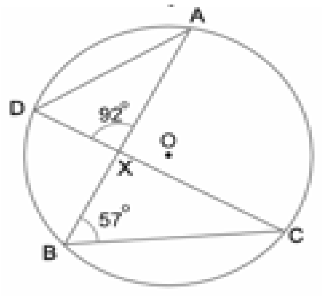
So $\angle VZW + 72° + 38° = 180°$

$\Rightarrow \angle VZW + 110° = 180°$

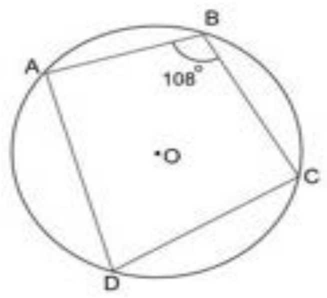
$\Rightarrow \angle VZW = 70°$

Exercise

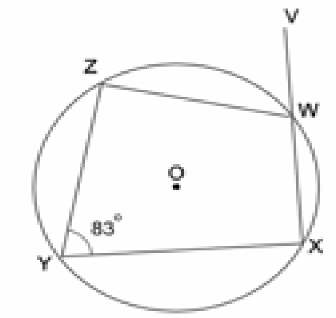
1. A, B, C and D are points on the circumference of a circle, center O.
   Chords AB and CD intersect at the point X
   $\angle AXD = 92°$ and $\angle CBA = 57°$
   What is the size of $\angle DAX$?



2. Refer to diagram below, ABCD is a cycle quadrilateral drawn inside a circle center O, $\angle ABC = 108°$. What is the size of $\angle ADC$

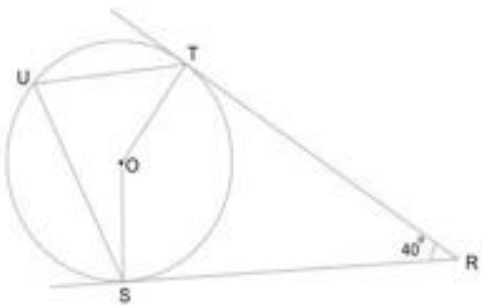


3.

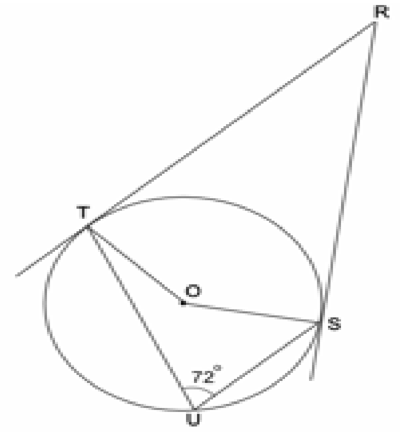


   WXYZ is a cycle quadrilateral drawn inside a circle center O and V is on the line XW extended. $\angle XYZ = 83°$
   What is the size of $\angle VXW$?

4. RS and RT are tangents to the circle center O. Given $\angle STR = 40°$. Find $\angle SUT$?



5. RS and RT are tangents to the circle center O, ∠SUT = 72°. What is the size of ∠SRT?



Review Answer

1. 31°
2. 72°
3. 83°
4. 70°
5. 36°

2.10 Arc length, area of Sector and the Segment of Circle

Radian

One radian is defined as the angle subtended from the center of a circle which intercepts an arc equal in length to the radius of the circle. The radian, denoted by the symbol rad, is the SI unit for measuring angles, and is the standard unit of angular measure used in many areas of mathematics

2.10.1 Conversion between radians and degrees

Conversion from radian to degree

As stated, one radian is equal to $\frac{180^\circ}{\pi}$. Thus, to convert from radians to degrees, multiply by $\frac{180^\circ}{\pi}$

$(\textit{value need to be converted})\ \text{x}\ \frac{180^\circ}{\pi}$

2.10.2 Conversion from degree to radian

Conversely, to convert from degrees to radians, multiply by $\frac{\pi}{180^\circ}$

$(\textit{value need to be converted})\ \text{x}\ \frac{\pi}{180^\circ}$

Example

Convert the following values to radian:

i. 120° ii. 147.2° iii. 27°52' iv. $27\frac{2^\circ}{3}$

Solution:

i. $120^\circ = 120 \text{ x } \frac{\pi}{180^\circ}$ radian

$= \frac{2\pi}{3}$ rad or 2.0947 rad

ii. $147.2^\circ = 147.2 \text{ x } \frac{\pi}{180^\circ}$ radian

$= 2.569$ rad

iii. 27°52’ $\Longrightarrow$ = 52’ convert to degree

60’ = 1°

$\frac{52'}{60'} = 0.86°$ so, 27°52’ = 27° + 0.86°

= 27.86°

Convert = 27.86° to rad

$= 27.86° \text{ x } \frac{\pi}{180^{\circ}}$

= 0.4862 rad

iv. $27\frac{2}{3}^{\circ}$ to radian

$\frac{(27x3)+2}{3} = \frac{83}{3}$

$\frac{83}{3} \text{ x } \frac{\pi}{180^{\circ}} = 0.4828 \text{ rad}$

Example

Convert the following values to degree:

i. 1.047 rad ii. $\frac{2\pi}{4}$ rad iii. π rad

Solution

i. $1.047 \text{ rad} = 1.047 \text{ x } \frac{180^{\circ}}{\pi}$

$= 60°$

iii. $\pi \text{ rad x } \frac{180^{\circ}}{\pi}$

$= 180°$

ii. $\frac{2\pi}{4} \text{rad x } \frac{180^{\circ}}{\pi} = 90°$

2.10.3 Circumference of a circle

Circumference = π

$= 2\pi r$

Arc Length

Arc length, s = $r\theta$, *where* θ must be in radian

2.10.4 Area of sector of circle

Area of sector = $\frac{n}{360^\circ}\pi r^2$

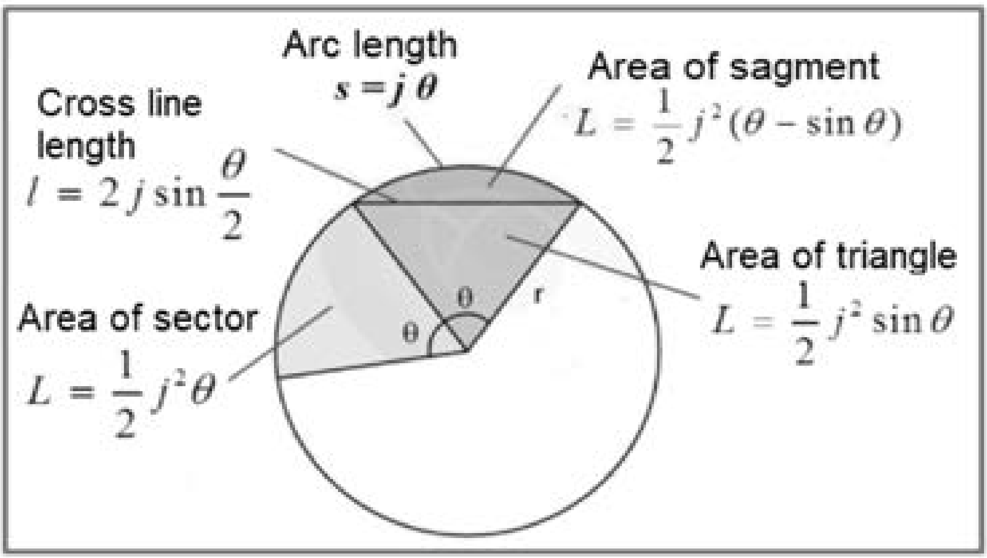
Where, n is angle of the sector in degree

Or

Area of sector = $\frac{1}{2}r^2\theta$

Where, θ is angle of the sector in radian.

The formula of properties of circle can be simplify by:



$j = r =$ radius, *θ must be in radian & θ with trigonometry term must be in degree*

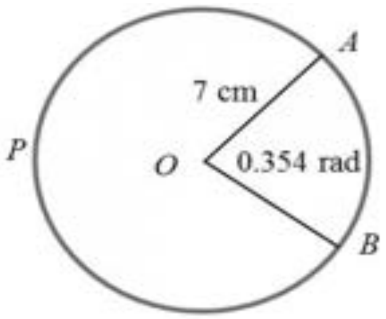
Area of segment

Area of segment = area of sector - area of triangle

Example

From the diagram above, find

i. Minor arc length AB
ii. Major arc length APB



Solution

i. Minor arc length AB, s = $j\theta$

$$= 7 \times 0.354\text{rad}$$
$$= 2.478 \text{ cm}$$

ii. Major arc length APB, s = $j\theta$

Reflects angle AOB = $2\pi - 0.354$ rad

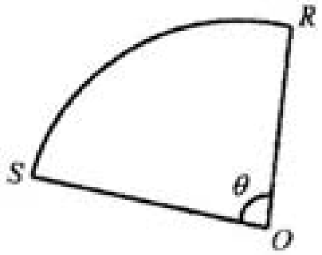
$$= 6.284 - 0.354$$
$$= 5.93 \text{ rad}$$

s =7 x 5.93rad

s = 41.51 cm

Example

The diagram below shows the ROS sector centered at O



Given that the radius OR is 9 cm and the angle $\theta = 1.8$ rad. Find the area of sector SOR

Solution

Method (i)

$$L = \frac{1}{2} j^2\theta$$

$$L = = \frac{1}{2}(9)^2(1.8)$$

$$= 72.9cm^2$$

Method (ii)

convert $\theta = 1.8$ rad

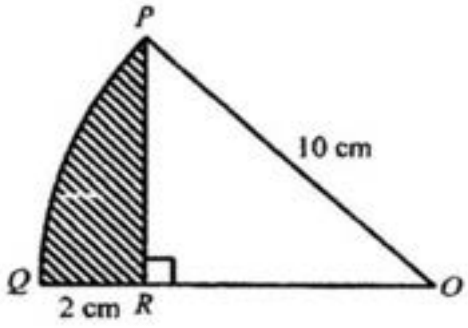
$$1.8 \times \frac{180^\circ}{\pi} = 103.13^\circ$$

Area of sector $L = \frac{\theta}{360^\circ} x(\pi)x(9)^2$

$$= L = \frac{103.13^\circ}{360^\circ} x(\pi)x(9)^2$$

$$= 72.89cm^2$$

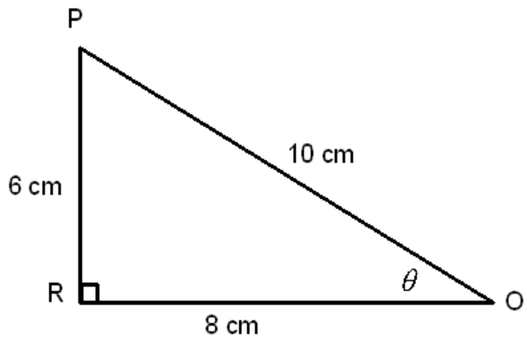
Example



The figure shows a OPQ sector, centered at O with radius of 10cm. Find:

i. ∠POQ in radian
ii. Area of shaded sector

Solution



i.
$$\cos\theta = \frac{8}{10} = 0.8$$
$$\theta = 36.87^\circ = 36.87^\circ\left(\frac{\pi}{180}\right) = 0.64 rad$$
$$\angle POQ = 0.64\ rad$$

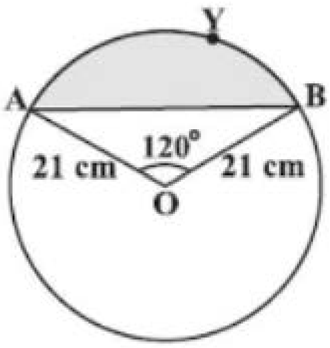
ii.
$$L1 = \frac{1}{2}j^2\theta \text{ (Area of sector OPQ)}$$
$$= \frac{1}{2}(10)^2(0.64)$$
$$= 32$$

$$L_2 = \frac{1}{2}dt \quad \text{(triangle)}$$
$$= \frac{1}{2}(8)(6)$$
$$= 24$$

So, $L = L_1 - L_2$
$= 32 - 24$
$= 8$, Area of shaded sector = $8cm^2$

Example

Find the area of the segment AYB shown in figure, if radius of the circle is 21 cm and ∠AOB = 120°. Use π = 3.142)



Solution

Convert 120° to radian $=120° \text{ x } \dfrac{\pi}{180^{\circ}} = 2.094$ rad

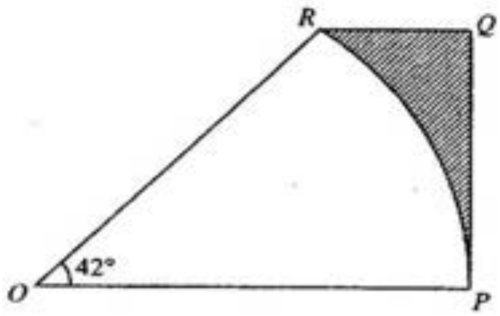
Area of sagment L $= \dfrac{1}{2}(j)^2(\theta - \sin\theta)$

$= \dfrac{1}{2}(21)^2(2.094 - \sin 120^{\circ})$

$= 270.77cm^2$

Example

The figure below shows the OPR sector centered at O and having a radius of 12cm. Given OPQR is a trapezoid.



Calculate:

i. Area of sector OPR
ii. Area of shaded diagram

Solution

i. Area of sector OPR

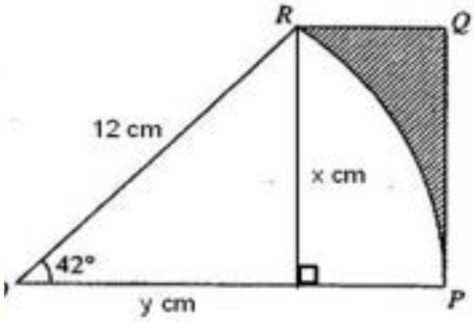
$$L = \frac{1}{2}j^2\theta$$

$$L = \frac{1}{2}(12)^2(42x\frac{\pi}{180^{\circ}})$$

$$L = \frac{1}{2}(12)^2(0.73rad)$$

$$L = 52.56cm^2$$

ii. Area of shaded diagram



$$\sin 42^\circ = \frac{x}{12}$$
$$12(\sin 42) = x$$
$$x = 8.03cm$$
$$\cos 42^\circ = \frac{y}{12}$$
$$12(\cos 42) = y$$
$$y = 8.92cm$$
$$L_1 = \frac{1}{2}(a+b)c$$
$$L_1 = \frac{1}{2}((12-8.92)+12)(8.03)$$
$$L_1 = \frac{1}{2}(3.08+12)(8.03)$$
$$L_1 = 60.55cm^2$$

$$L = L_1 - L_2$$
$$L = 60.55 - 52.56$$
$$L = 7.99cm^2$$

So, area of shaded diagram = $7.99cm^2$

Exercise

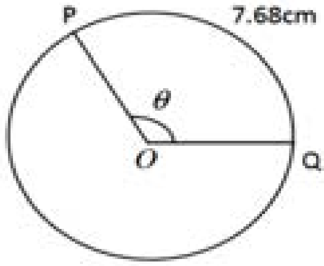
1. Convert the following values to radian

   i. $15°$ ii. $90°$ iii. $315°$ iv. $300°$

2. Convert the following values to degree

   i. 0.5 rad ii. $\frac{2}{3}\pi$ rad iii. 4.562 rad iv. $(2\pi - 1.5)$rad

3. The diagram below shows a circle with sector POQ having a radius 6cm. Given the length of the minor arc PQ is 7.68cm. Find the value of θ in radian

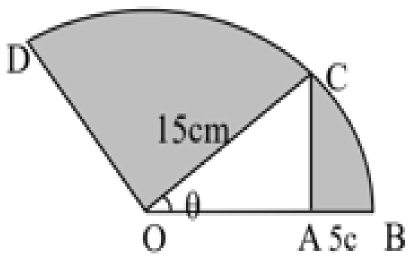


4. The diagram shows a circle with center O and diameter PR. Given that PR = 20cm and angle POQ = 1.2 rad. Find

   i. The area of the POQ minor sector

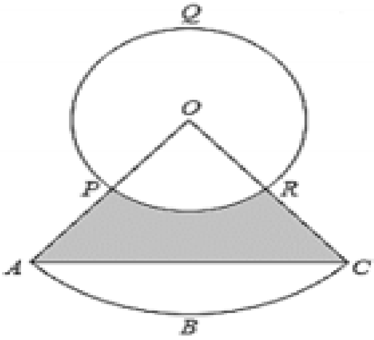
   ii. QOR sector area.

5. The diagram shows the sector of a circle centered at O and AOC is a right triangle.

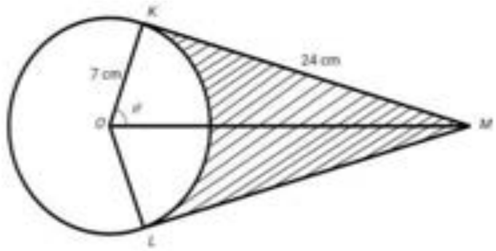


Given OC = OD = 15cm. Calculate the value of θ in radian, the the area of the shaded region

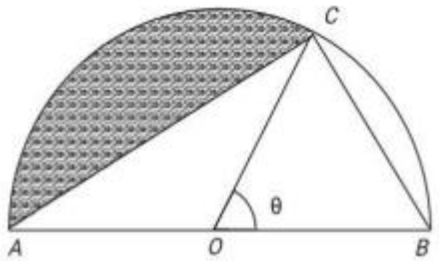
6. The figure below shows a circle PQR centered at O and having a radius of 10cm. The OABC is a sector of a circle centered at and having a radius of 22 cm. Given that the arc length ABC is 35cm, Find
   i. PQR in radian
   ii. Perimeter in cm of the shaded area.
   iii. Area in cm of shaded area



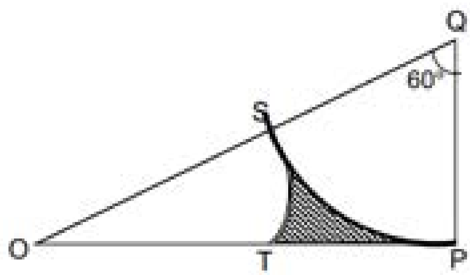
7. The figure shows a circle of radius 7cm with center O. KM and LM are two-line tangent to the circle at K and L. Given KM = 24cm, Calculate
   i. The value of θ in radian
   ii. The perimeter of shaded region
   iii. The area of shaded region



8. In the figure below, AOBC is an O centered semi-circle with a radius of 5cm. The OBC tringle is an equilateral triangle, calculate:
   i. The value of θ in radian
   ii. Perimeter in cm of shaded area
   iii. The area of shaded area.



9. The figure below shows the right angled triangle OPQ and the sectors SOT and PQS centered at O and Q, respectively. Given that OS = SQ and the perimeter of the shaded area is 16.12 cm. Calculate the value of the radius QS



Review Answer

1. i. $\frac{\pi}{12}$ rad   ii. $\frac{\pi}{2}$ rad   iii. 1.75π rad   iv. 1.67π rad
2. i. 28.65°   ii. 120°   iii. 261.38°   iv. 274.06°
3. 1.28 rad
4. i. 60cm^2 ii. 97.1cm^2
5. i. 0.8412   ii. 126.732
6. i. 1.591 rad   ii. 102.75   iii. 162.40 cm^2
7. i. 1.287rad   ii. 66.018cm   iii. 104.937 cm^2
8. 1. 1.047rad   ii. 37.2136cm   iii. 15.362 cm^2
9. 6.98 cm

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